DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

CALCULATORS ARE NOT PERMITTED

YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK

SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT

Name and section: _____

GSI's name: _____

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. Find all first partial derivatives of the following functions:

   (a) (5 points)

   $$f(x, y) = \sin(x^2 y).$$

   Solution:

   (b) (15 points)

   $$f(x, y, z) = zy \ln(xy + 2).$$

   Solution:

2. Let $f(x, y) = x^2 + xy - 2x - 2y + 2$

   (a) (15 points) Find all the possible relative maxima/minima using the first derivative test.

   Solution:

   (b) (10 points) Use the second derivative test to determine the nature of each such point.

   Solution:

3. (20 points) Calculate the following double integral

$$\iint_R (x + y^2) dx dy,$$

where R is the region enclosed by $y = x$, $y = 1/x$ and $x = 2$.

Solution:

4. (20 points) Determine the area enclosed by the graph $y = \sin(x) + 1$ and $y = \cos(x)$ between $x = -\pi/6$ and $x = \pi/3$. Simplify your answer as much as possible.

   Solution:

5. (25 points) The total cost to produce x large jewellery-making kits and y small ones is given by

$$C(x, y) = 2x^2 + 6y^2 + 4xy + 10.$$

If a total of ten kits must be made, how should production be allocated so that total cost is minimized? Use the method of Lagrange Multipliers to solve this problem. Be sure to justify why it is a minimum.

Solution:

END OF EXAM