MATH 16B MIDTERM 1 (001) 12.10PM - 1PM
PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

CALCULATORS ARE NOT PERMITTED

YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK

SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS FINISHED

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT

Name and section: ______________________________

GSI's name: ______________________________

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This exam consists of 5 questions. Answer the questions in the spaces provided.

1. Find all first partial derivatives of the following functions:

(a) (10 points)

$$f(x,y) = \ln(x\sqrt{y+1}).$$

Solution:

$$f(x,y) = \ln(x\sqrt{y+1}) = \ln(x) + \frac{1}{2}\ln(y+1) \Rightarrow$$

$$\frac{\partial f}{\partial x} = \frac{1}{x}, \quad \frac{\partial f}{\partial y} = \frac{1}{2(y+1)}$$

(b) (15 points)

$$f(x,y,z) = \frac{x^2y}{\sec(xz)}.$$

Solution:

$$f(x,y,z) = \frac{x^2y}{\sec(xz)} = x^2y\cos(xz) \Rightarrow$$

$$\frac{\partial f}{\partial x} = 2xy\cos(xz) - x^2yz\sin(xz)$$

$$\frac{\partial f}{\partial y} = x^2\cos(xz)$$

$$\frac{\partial f}{\partial z} = -x^3y\sin(xz)$$

2. (25 points) Determine the equation of the tangent line to

$$y = \cos(2x)\tan(x)$$

at $x = -\pi/6$.

Solution:

$$y\left(\frac{-\pi}{6}\right) = \cos\left(-\frac{2\pi}{6}\right)\tan\left(\frac{-\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right)\cdot\left(-\tan\left(\frac{\pi}{6}\right)\right)$$

$$= \frac{1}{2}\cdot\left(\frac{-1}{\sqrt{3}}\right) = \frac{-1}{2\sqrt{3}}$$

$$\frac{dy}{dx} = -2\sin(2x)\tan(x) + \cos(2x)\sec^2(x)$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=\frac{-\pi}{6}} = -2\sin\left(\frac{-\pi}{3}\right)\tan\left(\frac{-\pi}{6}\right) + \cos\left(\frac{-\pi}{3}\right)\cdot\frac{1}{\cos^2\left(\frac{-\pi}{6}\right)}$$

$$= -2\sin\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{6}\right) + \frac{\cos\left(\frac{\pi}{3}\right)}{\left(\cos\left(\frac{\pi}{6}\right)\right)^2} = -2\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{3}} + \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= -1 + \frac{2}{3} = -\frac{1}{3}$$

$\Rightarrow$ Tangent has equation $\left(y + \frac{1}{2\sqrt{3}}\right) = \frac{-1}{3}\left(x + \frac{\pi}{6}\right)$

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3. Let $f(x,y) = 3x^2 + 2y^3 - 18xy + 5$

(a) (15 points) Find all the possible relative maxima/minima using the first derivative test.

Solution:

$$\frac{\partial f}{\partial x} = 6x - 18y = 0$$

$$\frac{\partial f}{\partial y} = 6y^2 - 18x = 0$$

$\Rightarrow x = 3y \Rightarrow 6y^2 - 54y = 0$

$\Rightarrow y = 0$ or $y = 9$

$\Rightarrow x = 0$ or 27

$\Rightarrow (0,0), (27,9)$ only possible relative max/min

(b) (10 points) Use the second derivative test to determine the nature of each such point.

Solution:

$$\frac{\partial^2 f}{\partial x^2} = 6, \quad \frac{\partial^2 f}{\partial y^2} = 12y, \quad \frac{\partial^2 f}{\partial x \partial y} = -18$$

$\Rightarrow D(x,y) = 72y - 324$

$D(0,0) = -324 < 0 \Rightarrow$ saddle

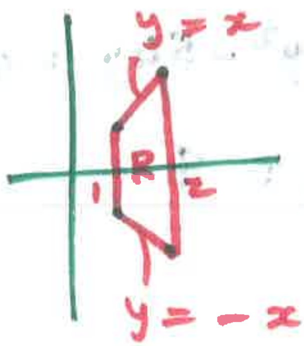
$D(27,9) > 0$ and $\frac{\partial^2 f}{\partial x^2}(27,9) = 6 > 0 \Rightarrow$ relative min

4. (25 points) Calculate the following double integral

$$\iint_R (x^2y^2+1)dxdy,$$

where R is the four-sided region with corners $(1,1)$, $(1,-1)$, $(2,2)$ and $(2,-2)$.

Solution:



$$\int_{-x}^{x} x^2y^2+1 \, dy = \frac{x^2y^3}{3}+y \Big|_{-x}^{x} = \frac{2}{3}x^5+2x$$

$$\Rightarrow \iint_R x^2y^2+1 \, dxdy = \int_1^2 \frac{2}{3}x^5+2x \, dx$$

$$= \frac{2}{18}x^6+x^2 \Big|_1^2$$

$$= \frac{1}{9}\cdot 64+4-\frac{1}{9}-1$$

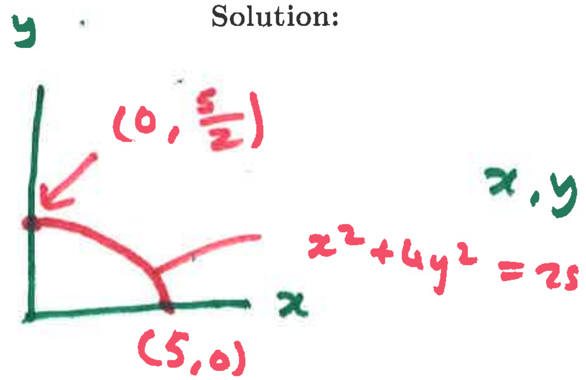
$$= 7+3 = 10.$$

5. (25 points) A company can make two products, A and B. If they make x units of A and y units of B, then they operate under the constraint:

$$x^2 + 4y^2 = 25$$

Suppose that it costs the company 3 dollars to make each unit of A and 8 dollars for each unit of B. Determine the minimal operating cost. Use the method of Lagrange Multipliers to solve this problem. Be sure to justify why it is a minimum.

Solution:



$x, y \geq 0$

$f(x,y) = 3x + 8y$

$g(x,y) = x^2+4y^2-25$

$$F(x,y,\lambda) = 3x + 8y + \lambda x^2 + 4\lambda y^2 - 25\lambda$$

$\Rightarrow \dfrac{\partial F}{\partial x} = 3 + 2\lambda x = 0$

$\dfrac{\partial F}{\partial y} = 8 + 8\lambda y = 0$

$\Rightarrow \lambda = \dfrac{-3}{2x} = \dfrac{-8}{8y} \Rightarrow y = \dfrac{2}{3}x$

$\dfrac{\partial F}{\partial \lambda} = x^2+4y^2-25 = 0 \Rightarrow x^2 + \dfrac{16}{9}x^2 = 25$

$\Rightarrow x^2 = 9 \Rightarrow x = 3 \Rightarrow y = 2$

$f(3,2) = 25$

$f(0, 5/2) = 20$

$f(5,0) = 15$

$\Rightarrow$ Costs are minimized when $x = 5$, $y = 0$.

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