DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

CALCULATORS ARE NOT PERMITTED

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT

THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM

Formulae

$$e^x = 1 + x + \tfrac{x^2}{2} + \tfrac{x^3}{6} + \cdots$$

$$\sin x = x - \tfrac{x^3}{3!} + \tfrac{x^5}{5!} - \tfrac{x^7}{7!} + \tfrac{x^9}{9!} - \cdots$$

$$\cos x = 1 - \tfrac{x^2}{2!} + \tfrac{x^4}{4!} - \tfrac{x^6}{6!} + \tfrac{x^8}{8!} - \cdots$$

$$\tfrac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + \cdots$$

$$\ln(1+x) = x - \tfrac{x^2}{2} + \tfrac{x^3}{3} - \tfrac{x^4}{4} + \tfrac{x^5}{5} - \cdots$$

Name and section: ______________________________

GSI's name: ______________________________

This exam consists of 10 questions. Answer the questions in the spaces provided.

1. (25 points) Find the points (x, y) which maximise the function y^2x, subject to the constraint $y^2 + 4xy = 1200$. You may assume, without justification, that a maximum exists.

   Solution:

2. (25 points) Let R be the region enclosed by $y = 1/x^2$, $y = 1/x$ and $x = 2$ in the first quadrant. Determine the following double integral

$$\iint_R \frac{x}{y}\, dxdy$$

Solution:

3. (25 points) Find the general solution to the following differential equation:

$$x^2y' = y\cos(1/x) + y$$

Hence, or otherwise, find solutions with the following initial conditions: $y(1/\pi) = 0$ and $y(1/\pi) = -1$.

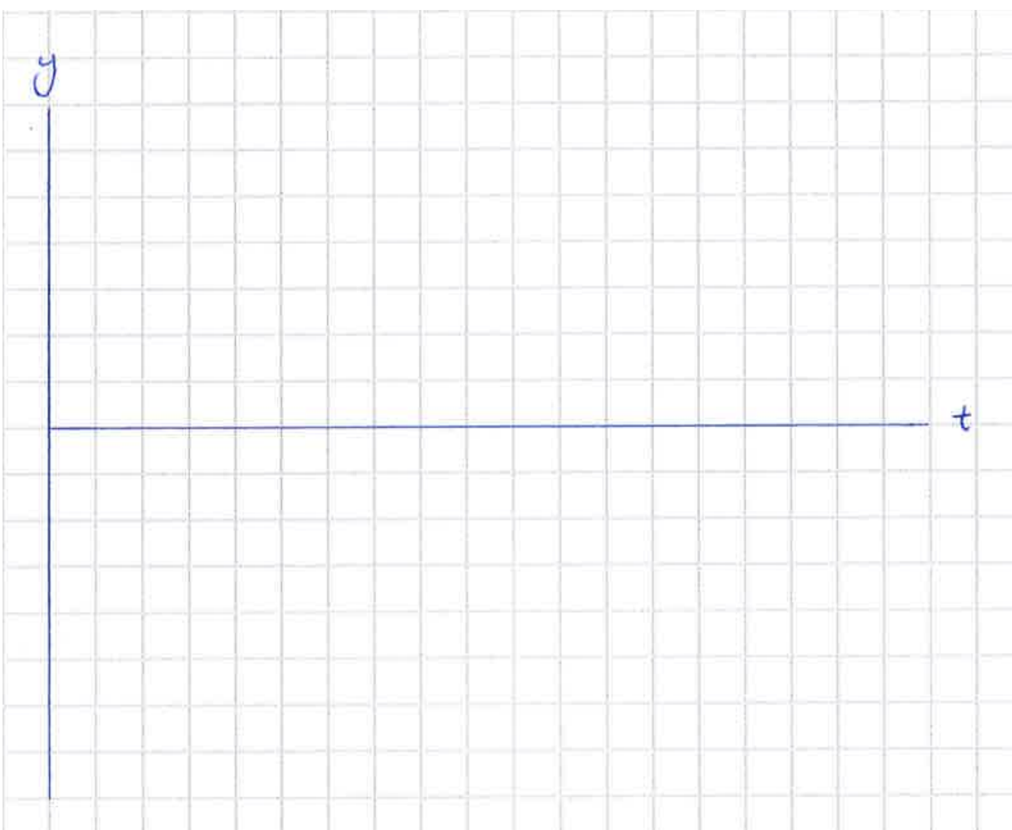
Solution:

4. (25 points) A company expects to have a constant income rate for the next ten years. They will invest it in a saving account with 50% annual interest rate. If they want to have \$10000000 in the account after ten years, what must their income rate be? You do not need to simplify your answer.

   Solution:

5. (25 points) Consider the differential equation $y' = \cos(y)$. Graph the solutions with the following initial conditions: $y(0) = -\pi/4$, $y(0) = \pi/2$ and $y(0) = 3\pi/4$. Be sure to label each solution carefully.

Solution:



6. (a) (15 points) Using the integral test, determine whether

$$\sum_{k=1}^{\infty} \frac{1}{k^3}$$

is convergent or divergent. Be sure to check all the hypotheses of the integral test.

Solution:

(b) (10 points) Using this, or otherwise, determine if the infinite series

$$\sum_{k=1}^{\infty} \frac{\sin^2(k)}{k^3}$$

is convergent or divergent.

Solution:

7. (25 points) Determine the Taylor series of the following functions. Write at least the first five non-zero terms of the sum to illustrate the general pattern.

   (a)
$$f(x) = e^{\frac{x}{2}}$$

   Solution:

   (b)
$$f(x) = \ln(1 - 2x^2)$$

   Solution:

8. (25 points) Let X be a continuous random variable on $[0, \pi/2]$ with probability density function $f(x) = \sin(x)$. Calculate the cumulative distribution function of X. Calculate $Var(X)$.

9. (25 points) Let Z be the standard normal random variable. Assume that

$$Pr(|Z| \leq \frac{1}{2}) = 0.4.$$

If X is a normal random variable with variance 4 and such that $Pr(X \geq 1) = \frac{1}{2}$, calculate

$$Pr(X \leq 2).$$

Solution:

10. (25 points) Let X be a discrete random variable which can take value any non-negative integer $0, 1, 2, 3, \ldots\ldots$ Assume that

$$Pr(X = n) = \frac{2}{3^{n+1}},$$

for any n, a non-negative integer. Calculate the probability that X is even. What is the expected value of X?

Solution:

END OF EXAM