MATH 16A FINAL EXAM (PRACTICE 3)
PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

CALCULATORS ARE NOT PERMITTED

YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT

THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM

Name and section: ____________________

GSI's name: ____________________

This exam consists of 10 questions. Answer the questions in the spaces provided.

1. Calculate the following derivatives (you do not need to use limits):

   (a)
   $$\frac{d}{dx}(2^x - x^2)$$

   Solution:

   (b)
   $$\frac{d}{du}(u^{1/3}\ln(u))$$

   Solution:

   (c)
   $$\frac{d^2}{dx^2}(\ln(\frac{(x-1)4^x}{x+1}))$$

   Solution:

2. Calculate the following integrals:

(a)

$$\int \sqrt{2x-1}dx$$

Solution:

(b)

$$\int \frac{2^x}{2^x+1}dx$$

Solution:

(c)

$$\int_1^2 \frac{\sqrt[3]{\ln(x)+3}}{x}dx$$

Solution:

3. Using the limit definition, calculate the derivative of $f(x) = \frac{1}{\sqrt{x}}$.

   Solution:

4. A product is being sold. The supply equation is given by

$$p = q^2 + 2q + 1.$$

The demand equation is given by

$$p = \frac{1000}{q+1}.$$

(a) Calculate the elasticity at the equilibrium.
Solution:

(b) Calculate the consumer surplus.
Solution:

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5. Find the equation of the tangent line at $(2, 1)$ of the following curve:

$$3(x^2 + y^2)^2 = 25(x^2 - y^2)$$

Solution:

6. Determine the relative extrema of the following function:

$$f(x) = xe^{(x^2-3x)}.$$

Are there any absolute extrema?

Solution:

7. A drink will be packaged in cylindrical cans with volume 40in^3. The top and bottom of the can cost 4 cents per square inch. The sides cost 3 cents per square inch. Determine the dimensions of the can which minimize costs.

   Solution:

8. Sketch the following curve. If they exist, be sure to indicate relative extrema and inflection points. Show your working on this page and draw the graph on the next page.

$$y = \frac{3x}{x-2}$$

Solution:

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9. Let $f(x) = x^5 - 2\ln((x+20)^3)$ and $g(x) = x^3 - 6\ln(x+20)$. Calculate the area of the region bounded by $y = f(x)$ and $y = g(x)$.

   Solution:

10. Two cars are travelling directly towards each other on a straight road. The first car is travelling at 3 metres per second. The second car is travelling at 6 metres per second. When they are 6 metres apart they simultaneously apply the brakes. The first car decelerates at a constant rate of 2 metres per second per second. The second car decelerates at a constant rate of 4 metres per second per second.

    (a) (15 points) How long after applying the brakes will the cars collide? Carefully justify your answer. Hint: If t is the time in seconds after they both apply the brakes, first calculate $s_1(t)$ and $s_2(t)$, position functions for the first and second car respectively.

    Solution:

    (b) (5 points) Determine the velocity of each car when they collide.

    Solution:

END OF EXAM