

Math 125 A – Fall 2013
Homework 3: Due Wednesday, September 25

Required Problems

Problem 1: Decide whether the following statements are **T**True or **F**False. Circle the right answer. You don't need to justify your answers.

T F There is $\Gamma \subseteq \text{Sent}^P$ which is consistent, but such that for any other sentence $\varphi \notin \Gamma$, $\Gamma \cup \{\varphi\}$ is inconsistent.

T F If P is finite, there is a finite sequence $\varphi_0, \dots, \varphi_n \in \text{Sent}^P$ such that any other $\psi \in \text{Sent}^P$ is semantically equivalent to one of them.

T F If P is finite, then Sent^P is finite.

T F If $\Gamma \vdash \varphi \vee \psi$ then either $\Gamma \vdash \varphi$ or $\Gamma \vdash \psi$.

T F If Γ and Δ are satisfiable, then so is $\Gamma \cup \Delta$.

Problem 2: Prove soundness for the rules ($\rightarrow E$) and ($\rightarrow I$) from the previous homework.

That is, assume these two rules were actual rules. Then, as in the proof of soundness that we did in class, we should consider the case when the *last rule applied* was one of these. Prove these cases.

Problem 3: Suppose that $\theta \vdash \gamma$ and $\gamma \vdash \theta$. Show that if $\Gamma \vdash \varphi$, then $\Gamma \vdash \text{Subst}_\gamma^\theta(\varphi)$.

Recall the function $\text{Subst}_\gamma^\theta$ from HW1. *Hint:* Staying only on the syntactic side is not advisable.

Problem 4: Suppose that we eliminate the *Contr* rule. Show that the Completeness Theorem no longer holds.

Hint: Notice that this rule is the only one that lets you get something new or smaller in the right hand side of $\Gamma \vdash \varphi$. State precisely what this gives you, and how it establishes the result.