Math 125 A – Fall 2013

Homework 10: Due Friday, December 6

Problem 1: In class we showed that if Γ is complete, consistent and contains witnesses, then it's term model $\mathcal{M}$ is a model of Γ. The proof was by induction on formulas. Write down the statement proved by induction, and prove the cases when the formula $\varphi(x_1, ..., x_k)$ is of the form $R(u_1(x_1, ..., x_k), ..., u_m(x_1, ..., x_k))$ and of the form $\forall x \psi(x, x_1, ..., x_k)$.

Problem 2:

We say that $T \subseteq Sent_{\mathcal{L}}$ *contains term witnesses* if whenever $T \models \exists x \varphi(x)$, the there is a closed term t such that $T \models \varphi(t)$.

Let $\mathcal{L}$ is the language $\mathcal{L} = \{0, 1, +, \times\}$.

(a) Give an example of a $T \subseteq Sent_{\mathcal{L}}$ such that T is complete, constant and contains term witnesses. (Notice we are keeping the same language and not adding constants or anything.)

(b) Give a different example in the same language.