Math 125 A – Fall 2013

Homework 1: Due Wednesday, September 11

Required Problems

Problem 1: Consider the following generating system. Let $U = \{1, 2, 3, 4, 5, 6, 7\}$, and $\mathcal{F} = \{f, g\}$, where $g\colon U \to U$ is given by

$$g(1) = 3 \quad g(2) = 1 \quad g(3) = 3 \quad g(4) = 7 \quad g(5) = 5 \quad g(6) = 1 \quad g(7) = 4$$

and $f\colon U^2 \to U$ is given by

	1	2	3	4	5	6	7
1	3	6	3	6	2	6	1
2	7	2	1	1	2	1	3
3	6	6	6	1	2	1	3
4	1	4	4	4	4	4	7
5	4	5	2	5	7	3	4
6	1	7	6	1	2	1	2
7	7	6	1	5	5	1	5

Interpret the diagram as follows. If $m, n \in U$, to calculate the value of $f(m, n)$, go to row m and column n. For example, $f(1, 2) = 6$.
a. Determine the set C generated from $B = \{5\}$ using $\mathcal{F}$. Justify your answer.
b. Determine the set C generated from $B = \{1\}$ using $\mathcal{F}$. Justify your answer.
c. Are any of these sets freely generated?

Problem 2: Let $U = \mathbb{N}^+$, $B = \{7, 13\}$, and $\mathcal{F} = \{g, f\}$ where $g\colon U \to U$ is given by $g(n) = 20\ n + 1$ and $f\colon U^2 \to U$ is given by $f(n, m) = 2^n(2m + 1)$. Show that the set generated from B using $\mathcal{F}$ is freely generated.

Problem 3: Define, for any $\theta, \gamma \in Sent^P$ a function $Subst^\theta_\gamma\colon Sent^P \to Sent^P$ (intuitively substituting θ for all occurrences of γ) recursively as follows.

- $Subst^\theta_\gamma(\mathsf{A}) = \begin{cases} \theta & \text{if } \gamma = \mathsf{A} \\ \mathsf{A} & \text{otherwise} \end{cases}$
- $Subst^\theta_\gamma(\neg\varphi) = \begin{cases} \theta & \text{if } \gamma = \neg\varphi \\ \neg Subst^\theta_\gamma(\varphi) & \text{otherwise} \end{cases}$
- $Subst^\theta_\gamma((\varphi \vee \psi)) = \begin{cases} \theta & \text{if } \gamma = (\varphi \vee \psi) \\ (Subst^\theta_\gamma(\varphi) \vee Subst^\theta_\gamma(\psi)) & \text{otherwise} \end{cases}$

Show by induction that if $v\colon P \to \{T, F\}$ is a truth assignment with $\overline{v}(\theta) = \overline{v}(\gamma)$, then $\overline{v}(\varphi) = \overline{v}(Subst^\theta_\gamma(\varphi))$ for every $\varphi \in Sent^P$.

Problem 4: Using the induction principle for generating systems, prove that on every sentence $\varphi \in Sent^P$ the number of left parenthesis is equal to the number of right parenthesis.