SYNTACTIC IMPLICATION

Basic Proofs:

- $\Gamma \vdash \varphi$ if $\varphi \in \Gamma \quad (Assume_{\mathcal{L}})$
- $\Gamma \vdash t = t$ for all $t \in Term_{\mathcal{L}} \quad (EqRefl)$

Proof Rules:

$$\frac{\Gamma \vdash \varphi \wedge \psi}{\Gamma \vdash \varphi} \ (\wedge EL) \qquad \frac{\Gamma \vdash \varphi \wedge \psi}{\Gamma \vdash \psi} \ (\wedge ER) \qquad \frac{\Gamma \vdash \varphi \quad \Gamma \vdash \psi}{\Gamma \vdash \varphi \wedge \psi} \ (\wedge I)$$

$$\frac{\Gamma \vdash \varphi}{\Gamma \vdash \varphi \vee \psi} \ (\vee IL) \qquad \frac{\Gamma \vdash \psi}{\Gamma \vdash \varphi \vee \psi} \ (\vee IR)$$

$$\frac{\Gamma \vdash \varphi \to \psi}{\Gamma \cup \{\varphi\} \vdash \psi} \ (\to E) \qquad \frac{\Gamma \cup \{\varphi\} \vdash \psi}{\Gamma \vdash \varphi \to \psi} \ (\to I)$$

$$\frac{\Gamma \cup \{\varphi\} \vdash \theta \quad \Gamma \cup \{\psi\} \vdash \theta}{\Gamma \cup \{\varphi \vee \psi\} \vdash \theta} \ (\vee PC) \qquad \frac{\Gamma \cup \{\psi\} \vdash \varphi \quad \Gamma \cup \{\neg\psi\} \vdash \varphi}{\Gamma \vdash \varphi} \ (\neg PC)$$

$$\frac{\Gamma \cup \{\neg\varphi\} \vdash \psi \quad \Gamma \cup \{\neg\varphi\} \vdash \neg\psi}{\Gamma \vdash \varphi} \ (Contr)$$

Equality Rules:

$$\frac{\Gamma \vdash \varphi^t_{\mathsf{x}} \quad \Gamma \vdash t = u}{\Gamma \vdash \varphi^u_{\mathsf{x}}} \ (= Sub)$$

Existential Rules:

$$\frac{\Gamma \vdash \varphi^t_{\mathsf{x}}}{\Gamma \vdash \exists \mathsf{x}\varphi} \ (\exists I) \qquad \frac{\Gamma \cup \{\varphi^{\mathsf{y}}_{\mathsf{x}}\} \vdash \psi}{\Gamma \cup \{\exists \mathsf{x}\varphi\} \vdash \psi} \ \text{if } \mathsf{y} \notin FreeVar(\Gamma \cup \{\exists \mathsf{x}\varphi, \psi\}) \ (\exists P)$$

Universal Rules:

$$\frac{\Gamma \vdash \forall \mathsf{x}\varphi}{\Gamma \vdash \varphi^t_{\mathsf{x}}} \ (\forall E) \qquad \frac{\Gamma \vdash \varphi^{\mathsf{y}}_{\mathsf{x}}}{\Gamma \vdash \forall \mathsf{x}\varphi} \ \text{if } \mathsf{y} \notin FreeVar(\Gamma \cup \{\forall \mathsf{x}\varphi\}) \ (\forall I)$$

Superset Rule:

$$\frac{\Gamma \vdash \varphi}{\Gamma' \vdash \varphi} \ \text{if } \Gamma' \supseteq \Gamma \ (Super)$$