Final (Practice) Solutions

There are 14 problems worth 10 points each. The exam will be scored out of 120, with a score exceeding 100% being possible. Show all of your work for full credit.

1. Find all real solutions to the equation.
$$|2x-5| = 9$$
Solution:
$$\begin{aligned} |2x-5| = 9 &\implies 2x-5 = \pm 9 \\ &\implies 2x = 5 \pm 9 \\ &\implies x = 7, -2 \end{aligned}$$

2. Find all real solutions to the equation.
$$3^{2x-1} = 2^{4x+1}$$
Solution:
$$\begin{aligned} 3^{2x-1} = 2^{4x+1} &\implies (2x-1)\ln 3 = (4x+1)\ln 2 \\ &\implies 2\ln 3x - 4\ln 2x = \ln 2 + \ln 3 \\ &\implies x = \frac{\ln 2 + \ln 3}{2\ln 3 - 4\ln 2} \end{aligned}$$

3. Determine whether the lines are parallel, perpendicular, or neither.
$$y = 2x + 3 \quad 2y - 4x - 5 = 0$$
Solution: The slope of the first line is $m = 2$. The slope of the second line is:
$$2y - 4x - 5 = 0 \implies 2y = 4x + 5 \implies y = 2x + \frac{5}{2}$$
also 2. So they are parallel.

4. Solve the inequality and express the solution using interval notation.
$$3x^2 - 3x < 2x^2 + 4$$
Solution:
$$\begin{aligned} 3x^2 - 3x < 2x^2 + 4 &\implies x^2 - 3x - 4 < 0 \\ &\implies (x-4)(x+1) < 0 \end{aligned}$$
The solution is $(-1, 4)$.

5. Solve the inequality and express the solution using interval notation.
$$\frac{x+2}{x+3} < \frac{x-1}{x-2}$$
Solution: Move everything to one side.
$$\begin{aligned} \frac{x+2}{x+3} < \frac{x-1}{x-2} &\implies \frac{x+2}{x+3} - \frac{x-1}{x-2} < 0 \\ &\implies \frac{(x+2)(x-2)}{(x+3)(x-2)} - \frac{(x-1)(x+3)}{(x+3)(x-2)} < 0 \\ &\implies \frac{x^2 - 4 - (x^2 + 2x - 3)}{(x+3)(x-2)} \\ &\implies \frac{-2x-1}{(x+3)(x-2)} \end{aligned}$$
Make a sign chart with $x = -3, -\frac{1}{2}, 2$ to conclude that $\left(-3, -\frac{1}{2}\right) \cup (2, \infty)$.

6. Find $f(g(h(x)))$.

$$f(x) = x^4 + 1 \quad g(x) = x - 5 \quad h(x) = \sqrt{x}$$

Solution:

$$\begin{aligned} f(g(h(x))) &= f(g(\sqrt{x})) \\ &= f(\sqrt{x} - 5) \\ &= (\sqrt{x} - 5)^4 + 1 \end{aligned}$$

7. Express the quadratic $f(x)$ in vertex form $a(x-h)^2 + k$. Classify the vertex as a maximum or a minimum.

$$f(x) = 3x^2 - 12x + 13$$

Solution:

$$\begin{aligned} 3x^2 - 12x + 13 &\implies 3(x^2 - 4x) + 13 \\ &\implies 3(x^2 - 4x + 4) + 13 - 12 \\ &\implies 3(x-2)^2 + 1 \end{aligned}$$

Since $a = 3 > 0$, the parabola opens upwards and the vertex $(2, 1)$ is a minimum.

8. Find all rational zeros of the polynomial and write in factored form.

$$P(x) = 4x^3 - 7x + 3$$

Solution: The rational root theorem gives the possible candidates as $x = \pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$. It is straightforward enough to tell that $x = 1$ is a root, so by long division of

$$\begin{array}{r@{}l} & \;4X^2 + 4X - 3 \\ \cline{2-2} X - 1 & \big)\; 4X^3 \qquad\quad - 7X + 3 \\ & \;\; -4X^3 + 4X^2 \\ \cline{2-2} & \qquad\qquad 4X^2 - 7X \\ & \qquad\quad -4X^2 + 4X \\ \cline{2-2} & \qquad\qquad\qquad -3X + 3 \\ & \qquad\qquad\qquad\;\; 3X - 3 \\ \cline{2-2} & \qquad\qquad\qquad\qquad\quad 0 \end{array}$$

Or by synthetic division:

$$\begin{array}{c|cccc} & 4 & 0 & -7 & 3 \\ 1 & & 4 & 4 & -3 \\ \hline & 4 & 4 & -3 & 0 \end{array}$$

Factor as

$$(4x^2 + 4x - 3) = (2x - 1)(2x + 3)$$

So

$$P(x) = 4x^3 - 7x + 3 = (x - 1)(2x - 1)(2x + 3)$$

9. If $\cos t = \frac{3}{5}$ and t is in quadrant IV, find $\cot t$.

Solution: We have the identity $\sin^2 t + \cos^2 t = 1$.

$$sin^2 t = 1 - \cos^2 t = \frac{16}{25} \implies \sin t = -\frac{4}{5}$$

where we take the negative root because we are in quadrant IV.

So $\cot t = \frac{\cos t}{\sin t} = \frac{\frac{3}{5}}{-\frac{4}{5}} = -\frac{3}{4}$.

10. Use an addition or subtraction formula to find the exact value of the expression.

$$\cos\left(\frac{11\pi}{12}\right)$$

Solution: $\frac{11\pi}{12}$ has to be a linear combination of

$$c_1\frac{\pi}{4} + c_2\frac{\pi}{3} = \frac{11\pi}{12} \implies 3c_1\frac{\pi}{12} + 4c_2\frac{\pi}{12} = \frac{11\pi}{12}$$

So $c_2 = 2, c_1 = 1$, and we have that

$$\frac{\pi}{4} + \frac{2\pi}{3} = \frac{11\pi}{12}$$

$$\begin{aligned}
\cos\left(\frac{11\pi}{12}\right) &= \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) \\
&= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{2\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{2\pi}{3}\right) \\
&= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)
\end{aligned}$$

11. Solve the equation.

$$\sin 2\theta - \sin\theta = 0$$

Solution:

$$\begin{aligned}
\sin 2\theta - \sin\theta = 0 &\implies 2\sin\theta\cos\theta - \sin\theta = 0 \\
&\implies \sin\theta(2\cos\theta - 1) = 0 \\
&\implies \sin\theta = 0 \text{ or } \cos\theta = \frac{1}{2} \\
&\implies \theta = \pi k \text{ or } \theta = \frac{\pi}{3} + 2\pi k, \frac{5\pi}{3} + 2\pi k
\end{aligned}$$

12. Verify the following identity.

$$\frac{1+\tan x}{1-\tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

Solution:

$$\begin{aligned}
\frac{1+\tan x}{1-\tan x} &= \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}} \\
&= \frac{\cos x + \sin x}{\cos x - \sin x}
\end{aligned}$$

13. Find two numbers whose sum is 34 and whose difference is 10.

Solution: The corresponding system is

$$\begin{cases} x + y &= 34 \\ x - y &= 10 \end{cases}$$

Add the two equations to cancel out the y to get $2x = 44 \implies x = 22$. Plug into the first equation to get $y = 12$.

14. Solve the following system or show that it has no solution.

$$\begin{cases} x+y+z &= 4 \\ x+3y+3z &= 10 \\ 2x+y-z &= 3 \end{cases}$$

Solution: We want to get this into upper triangular form so we can back substitute. We take -1 times the first equation and add it to the second, and -2 times the first equation and add it to the third.

$$\begin{cases} x+y+z &= 4 \\ +2y+2z &= 6 \\ -y-3z &= -5 \end{cases}$$

Swap equations 2 and 3.

$$\begin{cases} x+y+z &= 4 \\ -y-3z &= -5 \\ +2y+2z &= 6 \end{cases}$$

Add 2 times equation 2 to equation 3.

$$\begin{cases} x+y+z &= 4 \\ -y-3z &= -5 \\ -4z &= -4 \end{cases}$$

Back substitute to get $z = 1, y = 2, x = 1$. Always a good idea to plug into the original system of equations.