

Friday 1/23 (Week 1) §1.1 – 1.2

This week, we covered:

- Addition and multiplication of fractions (and subtraction and division).
- Sets, set notation, union and intersection.
- Interval notation.
- Absolute values.
- Exponents and Radicals.

1. (§1.1.30b) $\frac{3}{2} - \frac{5}{8} + \frac{1}{6}$.

Solution: Addition and subtraction are all about finding a common denominator, ideally the least common denominator. Since $2 = 2^1$, $8 = 2^3$, $6 = 2 \cdot 3$, we have the least common denominator is $2^3 \cdot 3 = 24$. This gives us

$$\frac{3}{2} - \frac{5}{8} + \frac{1}{6} = \frac{3(12)}{2(12)} - \frac{5(3)}{8(3)} + \frac{1(4)}{6(4)} = \frac{36}{24} - \frac{15}{24} + \frac{4}{24} = \frac{25}{24}$$

2. (§1.1.32a) Simplify

$$\frac{2}{\frac{2}{3}} - \frac{\frac{2}{3}}{2}$$

Solution: Dividing a fraction is the same as multiplying by its reciprocal.

$$\frac{2}{\frac{2}{3}} - \frac{\frac{2}{3}}{2} = 2 \left(\frac{3}{2} \right) - \frac{1}{3} = 3 - \frac{1}{3} = \frac{8}{3}$$

3. (§1.1.38a) Is the inequality true or false?

$$\frac{7}{11} \geq \frac{8}{13}$$

Solution: Convert to a common denominator so we can directly compare the numerators. Since 11 and 13 are both prime, we have the least common denominator as $11 \cdot 13 = 143$. So we have

$$\frac{7(13)}{11(13)} \geq \frac{8(11)}{13(11)} \implies \frac{91}{143} \geq \frac{88}{143}$$

So this inequality is true.

4. (§1.1.40d) Express the following statement as an inequality and also write in interval notation. w is positive and less than or equal to 17.

Solution: w is positive means $w > 0$. w is less than or equal to 17 is $w \leq 17$. So satisfying both parts of the statement gives us $0 < w \leq 17$. In interval notation this is $(0, 17]$. Note the use of the open and closed bracket.

5. (§1.1.39e) Express the following statement as an inequality and also write in interval notation. The distance from p to 3 is at most 5.

Solution: When we see distance we should think about absolute value. The presence of at most 5 means we must have “something ≤ 5 ”. Now we think about how to express the distance between p and 3, which is precisely $|p - 3|$. So $|p - 3| \leq 5$.

6. (§1 review 10) Evaluate the expression

$$5 - |10 - | - 4||$$

Solution:

$$5 - |10 - | - 4|| = 5 - |10 - 4| = 5 - 6 = -1$$

7. (§1.1.41 – 44) Given the sets

$$A = \{1, 2, 3, 4, 5, 6, 7\} \quad B = \{2, 4, 6, 8\} \quad C = \{7, 8, 9, 10\}$$

Compute the following quantities:

- a) $A \cup B$
- b) $A \cap B$
- c) $B \cup C$
- d) $B \cap C$
- e) $A \cup C$
- f) $A \cap C$
- g) $A \cup B \cup C$
- h) $A \cap B \cap C$

Solution:

- a) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- b) $A \cap B = \{2, 4, 6\}$
- c) $B \cup C = \{2, 4, 6, 7, 8, 9, 10\}$
- d) $B \cap C = \{8\}$
- e) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- f) $A \cap C = \{7\}$
- g) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- h) $A \cap B \cap C = \emptyset$

8. (§1.2.37b, 38b, 40a) Simplify each expression. Assume that all letters denote positive real numbers.

- a) $\sqrt{75} + \sqrt{48}$
- b) $\sqrt[3]{54} - \sqrt[3]{16}$
- c) $\sqrt[3]{x^4} + \sqrt[3]{8x}$

Solution:

- a) $\sqrt{75} + \sqrt{48} = \sqrt{25 \cdot 3} + \sqrt{16 \cdot 3} = 5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$
- b) $\sqrt[3]{54} - \sqrt[3]{16} = \sqrt[3]{27 \cdot 2} - \sqrt[3]{2^4} = 3\sqrt[3]{2} - 2\sqrt[3]{2} = \sqrt[3]{2}$
- c) $\sqrt[3]{x^4} + \sqrt[3]{8x} = xx^{1/3} + 2x^{1/3} = (2 + x)x^{1/3}$

9. (§1 Review 17 – 20) Simplify the expression.

- a) $(a^2)^{-3}(a^3b)^2(b^3)^4$
- b) $(3xy^2)^3(\frac{2}{3}x^{-1}y)^2$
- c) $\frac{x^4(3x)^2}{x^3}$

$$d) \left(\frac{r^2 s^{4/3}}{r^{1/3} s} \right)^6$$

$$e) \sqrt[3]{(x^3 y)^2 y^4}$$

$$f) \frac{8r^{1/2} s^{-3}}{2r^{-2} s^4}$$

$$g) \left(\frac{ab^2 c^{-3}}{2a^3 b^{-4}} \right)^{-2}$$

Solution:

$$a) (a^2)^{-3} (a^3 b)^2 (b^3)^4 = a^{-6} a^6 b^2 b^{12} = b^{14}$$

$$b) (3xy^2)^3 \left(\frac{2}{3}x^{-1}y\right)^2 = 27x^3 y^6 \frac{4}{9} x^{-2} y^2 = 12xy^8$$

$$c) \frac{x^4 (3x)^2}{x^3} = x(9x^2) = 9x^3$$

$$d) \left(\frac{r^2 s^{4/3}}{r^{1/3} s} \right)^6 = (r^{5/3} s^{1/3})^6 = r^{10} s^2$$

$$e) \sqrt[3]{(x^3 y)^2 y^4} = \sqrt[3]{x^6 y^2 y^4} = \sqrt[3]{x^6 y^6} = x^2 y^2$$

$$f) \frac{8r^{1/2} s^{-3}}{2r^{-2} s^4} = 4 \frac{r^{\frac{5}{2}}}{s^7}$$

$$g) \left(\frac{ab^2 c^{-3}}{2a^3 b^{-4}} \right)^{-2} = \left(\frac{b^6}{2a^2 c^3} \right)^{-2} = \frac{4a^4 c^6}{b^{12}}$$

Friday 1/30 (Week 2) §1.3 – 1.5

This week, we covered:

- Factoring
 - Factoring trinomials directly.
 - Factoring by grouping.
 - Factoring by the quadratic formula.
- Completing the square
- Adding, subtracting, multiplying and dividing polynomials.
- Simplifying rational functions (by factoring polynomials)
- Finding domains.
- Rationalizing the numerator/denominator.
- Solving equations (linear, quadratic, with radicals, absolute values, rational functions)

This is probably the most important week of the class as it is completely foundational for everything else we are going to do. A lot of the problems in this class or in calculus boils down to solving an equation. Solving an equation frequently requires factoring polynomials. Do LOTS of practice on your own until this becomes second nature.

I forgot to solve an equation involving an absolute value on Wednesday. I have added it to the notes and I do so here.

Example 0.1: Solving an equation with an absolute value

Solve the equation

$$|3x - 8| = 4$$

Solution: We have that $3x - 8$ can either be equal to 4 or -4 . So

$$3x - 8 = 4 \implies 3x = 12 \implies x = 4$$

$$3x - 8 = -4 \implies 3x = 4 \implies x = \frac{4}{3}$$

1. (§1.4.11, 13, 15) Find the domain of the following functions.

$$\sqrt{x+3} \quad \frac{x^2+1}{x^2-x-2} \quad \frac{\sqrt{x-2}}{x+3}$$

Solution: When it comes to finding domains, we typically want to avoid two things: 1) dividing by 0 and 2) taking the square root of a negative number.

- Since we are taking the square root of $x + 3$, we need to make sure it is nonnegative. So we need to solve the inequality $x + 3 \geq 0 \implies x \geq -3$. In interval notation this is $[-3, \infty)$.
- The numerator is a polynomial and it is defined for all real numbers. We require the denominator to be nonzero.

$$x^2 - x - 2 = 0 \implies (x - 2)(x + 1) = 0 \implies x = 2, -1.$$

So the domain is all real numbers except for 2, -1 . In interval notation this is $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$.

- We require $x \geq 2$ and $x \neq -3$. The second condition is redundant, so we have $x \geq 2$. In interval notation this is $[2, \infty)$.

2. (§1.4.89) Rationalize the denominator

$$\frac{2}{\sqrt{5} - \sqrt{3}}$$

Solution: We are going to multiply the denominator by its conjugate $\sqrt{5} + \sqrt{3}$ and exploit the difference of squares.

$$\frac{2}{\sqrt{5} - \sqrt{3}} = \frac{2(\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})} = \frac{2(\sqrt{5} + \sqrt{3})}{5 - 3} = \sqrt{5} + \sqrt{3}$$

Note that rationalizing the denominator is almost always going to make the numerator irrational, and vice versa.

Now I am going to spend most of the time factoring trinomials (polynomials with three terms) and in particular quadratics (polynomials of degree 2).

3. (§1.3.73, 74, 75, 77, 78, 103, 105, 108)) Factor each of the polynomials completely:

- a) $x^2 - 11x + 18$
- b) $x^2 + 4x - 5$
- c) $x^2 - 2x - 8$
- d) $3x^2 - 16 + 5$
- e) $5x^2 - 7x - 6$
- f) $2x^2 + 5x + 3$
- g) $2x^2 + 7x - 4$
- h) $8x^2 + 10x + 3$

Solution: Let's first start with polynomials where the leading coefficient is one.

- a) $x^2 - 11x + 18$

We have $(x + c)(x + d)$ where $cd = 18$ and $c + d = -11$. So the possible pairs are $(c, d) = (-1, -18), (-2, -9), (-3, -6)$. We do not need to consider the pairs of two positive integers since their sum is negative. We also do not need to consider the three pairs in the other order since c and d are interchangeable. The winning pair is $(c, d) = (-2, -9)$ so this is factored as

$$x^2 - 11x + 18 = (x - 2)(x - 9)$$

- b) $x^2 + 4x - 5$

We have $(x + c)(x + d)$ where $cd = -5$ and $c + d = 4$. The possible pairs to consider is $(-1, 5)$ and $(1, -5)$, and the former has the correct sum. So we have

$$x^2 + 4x - 5 = (x + 5)(x - 1)$$

- c) $x^2 - 2x - 8$

We consider the pairs $(-1, 8), (-2, 4), (-4, 2), (-8, 1)$. The third pair has the correct sum.

$$x^2 - 2x - 8 = (x - 4)(x + 2)$$

Now we move onto the case where the leading coefficient is not 1, but can only be factored one possible way (prime numbers).

d) $3x^2 - 16x + 5$

This must take the form $(3x+c)(x+d)$. We only need to consider the pairs $(-1, -5), (-5, -1)$. We do not need to consider the pairs $(1, 5)$ and $(5, 1)$ since the coefficient of the x term is negative. Unlike before, c and d are not interchangeable, so we need to consider $c = -1, d = -5$ and $c = -5, d = -1$

$$(3x - 1)(x - 5) = 3x^2 - 15x - x + 5 = 3x^2 - 16x + 5 \quad \checkmark$$

$$(3x - 5)(x - 1) = 3x^2 - 3x - 5x + 5 = 3x^2 - 8x + 5 \quad \text{X}$$

e) $5x^2 - 7x - 6$

This must take the form $(5x+c)(x+d)$. We need to consider the pairs $(1, -6), (2, -3), (3, -2), (6, -1)$ as well as the four pairs in the opposite order $(-6, 1), (-3, 2), (-2, 3), (-1, 6)$. Starting out, there is no substitute but to check all eight combinations. Over time you will gain experience and be able to do this “mentally” without doing all of these checks. For example, frequently you can discard the “extreme pairs” if the middle term has a coefficient that is not that large in absolute value. That’s why you need to practice a lot - there is no hard and fast rule when it comes to this, unlike using a formula.

$$(5x + 1)(x - 6) = 5x^2 - 29x - 6 \quad \text{X}$$

$$(5x + 2)(x - 3) = 5x^2 - 13x - 6 \quad \text{X}$$

$$(5x + 3)(x - 2) = 5x^2 - 7x - 6 \quad \checkmark$$

I will also take this chance to demonstrate the quadratic formula to find the two roots, which are $x = -\frac{3}{5}, 2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{7 \pm \sqrt{(-7)^2 - 4(5)(-6)}}{2(5)} = \frac{7 \pm \sqrt{49 + 120}}{10} = \frac{7 \pm 13}{10} = 2, -\frac{3}{5}$$

So we know the polynomial must be a multiple of $(x - 2)(x + \frac{3}{5})$, and since the leading coefficient is 5 we must multiply by 5. Let’s just do it on the second term to clear fractions and get the same result.

f) $2x^2 + 5x + 3$

It can take the form $(2x+c)(x+d)$. The choices of (c, d) can be $(1, 3)$ and the reverse pairs $(3, 1)$. No need to consider the negative pairs. So let’s check all 8 possibilities.

$$(2x + 1)(x + 3) = 2x^2 + 7x + 3 \quad \text{X}$$

$$(2x + 3)(x + 1) = 2x^2 + 5x + 3 \quad \checkmark$$

g) $2x^2 + 7x - 4$

It can take the form $(2x+c)(x+d)$. The choices of (c, d) can be $(-1, 4), (-2, 2)$ and the reverse pairs $(4, -1), (2, -2)$ since they are not interchangeable. No need to consider the negative pairs. So let’s check all 8 possibilities.

$$(2x - 1)(x + 4) = 2x^2 + 7x - 3 \quad \checkmark$$

$$(2x + 2)(x + 2) = \dots$$

h) $8x^2 + 10x + 3$

It can take the form $(8x + c)(x + d)$, $(4x + c)(2x + d)$. The choices of (c, d) can be $(1, 3)$ and $(3, 1)$. So let's check all 4 possibilities.

$$(8x + 1)(x + 3) = 8x^2 + 25x + 3 \quad \text{X}$$

$$(8x + 3)(x + 1) = 8x^2 + 11x + 3 \quad \text{X}$$

$$(4x + 3)(2x + 1) = 8x^2 + 10x + 3 \quad \checkmark$$

4. (§1.3.122) Factor the polynomial completely:

$$3x^3 - 27x$$

Solution: Sometimes we can factor out constants or powers of x just like in this case:

$$3x^3 - 27x = 3x(x^2 - 9)$$

and then what's left is typically a quadratic we need to factor using the above means. In this case we can recognize $x^2 - 9$ as the difference of squares, so we have

$$3x^3 - 27x = 3x(x + 3)(x - 3).$$

I wish I had more time, but let's solve some equations as we can in the time left.

5. (§1.5.106, 88, 113) Solve the following equations.

a) $\frac{y + 1}{y^2 + 1} = \frac{2}{y + 2}$

b) $\sqrt{3 + x} = \sqrt{x^2 + 1}$

c) $|3x - 10| = 29$

Solution:

a) Cross multiply to clear the denominators:

$$\frac{y + 1}{y^2 + 1} = \frac{2}{y + 2} \implies (y + 1)(y + 2) = 2(y^2 + 1) \implies y^2 + 3y + 2 = 2y^2 + 2 \implies y^2 - 3y = 0 \implies y(y - 3) = 0$$

So $y = 0, 3$. Always good to plug back in to see if it satisfies the original equation.

b) $\sqrt{3 + x} = \sqrt{x^2 + 1}$

$$\sqrt{3 + x} = \sqrt{x^2 + 1} \implies 3 + x = x^2 + 1 \implies x^2 - x - 2 = 0 \implies (x - 2)(x + 1) = 0$$

Plugging in $x = 2, -1$ gives us neither are extraneous solutions and both are valid.

c) $|3x - 10| = 29$

$$|3x - 10| = 29 \implies 3x - 10 = \pm 29 \implies 3x = 10 \pm 29 \implies 3x = 39 \text{ or } 3x = -19$$

So $x = 13, -\frac{19}{3}$.

Friday 2/06 (Week 3) §1.8 – 1.10

This week, we covered:

- Solving inequalities - linear, quadratic (in general polynomial), absolute values, and with rational functions.
- Distance and midpoint formulas.
- Circles.
- Graphing equations by plotting points.
- x and y intercepts.
- Lines
 - Slope of lines.
 - Vertical and horizontal lines.
 - Slope-intercept and point-slope form of lines.
 - Parallel and perpendicular lines.

1. (§1.8.41, 54, 68, 83) Solve the following inequalities.

a) $x^2 - 3x - 18 \leq 0$

b) $(x - 4)(x + 2)^2 > 0$

c) $\frac{x}{x+1} > 3x$

d) $\left| \frac{x-2}{3} \right| < 2$

Solution:

a) Factor

$$x^2 - 3x - 18 \leq 0 \implies (x - 6)(x + 3) \leq 0$$

and make a sign chart with $x = -3, 6$. The solution is $[-3, 6]$.

b) Make a sign chart at $x = 4, -2$. The result is $(4, \infty)$. Note that $(x + 2)^2$ is always positive so this does not affect the sign. This shows that signs do not necessarily have to alternate in the sign chart.

c) Move everything over to the LHS and combine with a common denominator

$$\begin{aligned} \frac{x}{x+1} > 3x &\implies \frac{x}{x+1} - \frac{3x(x+1)}{(x+1)} > 0 \\ &\implies \frac{x - 3x^2 - 3x}{x+1} > 0 \\ &\implies \frac{-3x^2 - 2x}{x+1} > 0 \\ &\implies \frac{x(-3x - 2)}{x+1} > 0 \end{aligned}$$

Make a sign chart with $x = -1, -\frac{2}{3}, 0$ to conclude $(-\infty, -1) \cup (-\frac{2}{3}, 0)$.

d)

$$\begin{aligned}\left|\frac{x-2}{3}\right| < 2 &\implies |x-2| < 6 \\ &\implies x-2 < 6 \text{ or } x-2 > -6 \\ &\implies x < 8, x > -4\end{aligned}$$

In interval notation this is $(-4, 8)$.

2. (§1.9.95) Show that the equation represents a circle, and find the circle and radius of the circle.

$$2x^2 + 2y^2 - 3x = 0$$

Solution: We need to rewrite this in the form

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

$$\begin{aligned}2x^2 + 2y^2 - 3x = 0 &\implies 2\left(x^2 - \frac{3}{2}x\right) + 2y^2 = 0 \\ &\implies 2\left(x^2 - \frac{3}{2}x\right) + 2y^2 = 0 \\ &\implies \left(x^2 - \frac{3}{2}x\right) + y^2 = 0 \\ &\implies \left(x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2\right) + y^2 = \left(\frac{3}{4}\right)^2 \\ &\implies \left(x - \frac{3}{4}\right)^2 + y^2 = \left(\frac{3}{4}\right)^2\end{aligned}$$

Center $\left(\frac{3}{4}, 0\right)$ and radius $\frac{3}{4}$.

3. (§1.9.87) Find the equation of a circle with center $(7, -3)$ that only touches but does not cross the x -axis. What are the circle's x and y -intercepts (if they exist)? Sketch the circle.

Solution: We know the equation must have the form $(x - 7)^2 + (y + 3)^2 = r^2$. The center at $(7, -3)$ is a point in the fourth (IV) quadrant, and for it to touch the x -axis $(7, 0)$ must also be a point. So the radius is $r = 3$ and the equation of the circle is

$$(x - 7)^2 + (y + 3)^2 = 9$$

4. (§1 Review 130) Find the equation of the horizontal line with y -intercept 5.

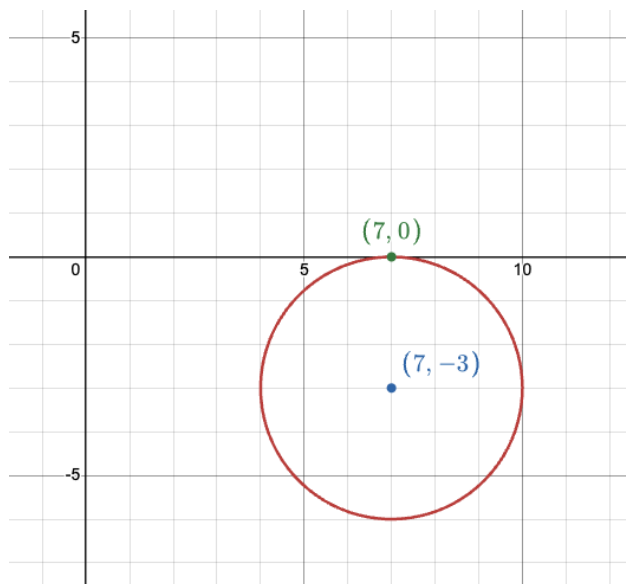
Solution: Don't get psyched out or confused with vertical lines. It's $y = 5$.

5. (§1 Review 131) Find the equation of the line that passes through the origin and is parallel to the line containing $(2, 4)$ and $(4, -4)$.

Solution: The slope of this line is $m = \frac{4 - (-4)}{2 - 4} = -4$, and it passes through the origin, so by point slope form this gives us

$$(y - 0) = -4(x - 0) \implies y = -4x$$

Actually, saying it passes through the origin also gives it the y -intercept 0 so $y = mx + b = -4x$.



6. (§1.1.78) Are the two lines parallel, perpendicular, or neither?

$$6y - 2x = 5 \quad 2y + 6x = 1$$

Solution: The slope of the first line is

$$6y - 2x = 5 \implies 6y = 2x + 5 \implies 6 = \frac{1}{3}x + \frac{5}{6}$$

The slope of the second line is

$$2y + 6x = 1 \implies 2y = -6x + 1 \implies y = -3 + \frac{1}{2}$$

The slopes are negative reciprocals of each other (or equivalently their product is -1), so they are perpendicular.

Friday 2/13 (Week 4) §2.1 – 2.2, 2.6

This week, we covered:

- Definition of functions $f(x)$ and evaluating functions at points in its domain.
- Domain of a function.
- Graphs of common functions - lines, power functions, root functions, reciprocal functions, absolute value function.
- Piecewise functions.
- The vertical line test.
- Transformations of functions and combining multiple transformations.

Today I am going to talk exclusively about transformations of functions and combining them. We already learned about the individual pieces:

1. Vertical shift
2. Horizontal shift
3. Reflecting across x or y axis
4. Vertical stretch/shrink
5. Horizontal stretch/shrink

Let me begin with a warning that application of the above transformations do not necessary commute. What do I mean by commute? Recall from the first day, when I introduced the commutative property for real numbers: $a + b = b + a$ and $ab = ba$. By do not necessarily commute I mean order matters.

Here is a concrete example. Suppose I have the graph $y = x^2$. If I apply the operations “shift to the left by 5” and “reflect across the y -axis”, I end up with

$$x^2 \rightarrow (x + 5)^2 \rightarrow (-x + 5)^2 = (x - 5)^2$$

If I apply these operators in reverse, first note that reflecting $y = x^2$ across the y -axis leaves it unchanged. So we end up with

$$x^2 \rightarrow x^2 \rightarrow (x + 5)^2$$

Let's start with a few warmups. This relies on you knowing the basic “parent” functions well and being able to recall them quickly.

1. Describe in words what is happening to $f(x)$.
 - a) $y = 5 + f(-x)$
 - b) $y = 3 - \frac{1}{2}f(x + 2)$
 - c) $y = 2 - f(5x)$
 - d) $y = 1 + f(\frac{1}{2}(x + 1))$

Solution:

- a) We have f is being reflected across the y axis, and then shifted up by 5 units.

$$f(x) \rightarrow f(-x) \rightarrow 5 + f(-x)$$

In this case order doesn't matter as we could have applied the transformations in reverse.

$$f(x) \rightarrow 5 + f(x) \rightarrow 5 + f(-x)$$

- b) **IMPORTANT NOTE IN TERMINOLOGY.** When I am talking about vertical stretching and shrinking, I will only say things like “stretched by a factor of c ” when $c > 1$ to denote $f(x) \rightarrow cf(x)$. When I am talking about shrinking such as $f(x) \rightarrow \frac{1}{2}x$, I will refer to this as “shrunk by a factor of $\frac{1}{2}$ ”, that is to say I only say things like “stretched by a factor of c ” when $0 < c < 1$. Back to the problem.

We have f is being shifted to the left by 2, then vertically shrunk by a factor of 2 (or stretched by a factor of $1/2$), reflected across the x -axis, and then shifted up 3 units.

$$f(x) \rightarrow f(x+2) \rightarrow \frac{1}{2}f(x+2) \rightarrow -\frac{1}{2}f(x+2) \rightarrow 3 - \frac{1}{2}f(x+2)$$

- c) We have f is being horizontally shrunk by a factor of 5, reflected across the x -axis, and then shifted up 2 units.

$$f(x) \rightarrow f(5x) \rightarrow -f(5x) \rightarrow 2 - f(5x)$$

- d) We have that $f(x)$ is being horizontally stretched by a factor of 2, then shifted to the left by 1, and then shifted up by 1.

$$f(x) \rightarrow f\left(\frac{1}{2}x\right) \rightarrow f\left(\frac{1}{2}(x+1)\right) \rightarrow 1 + f\left(\frac{1}{2}(x+1)\right)$$

2. Sketch the following. Label at least 3 points on the graph where the (x, y) coordinates are rational numbers.

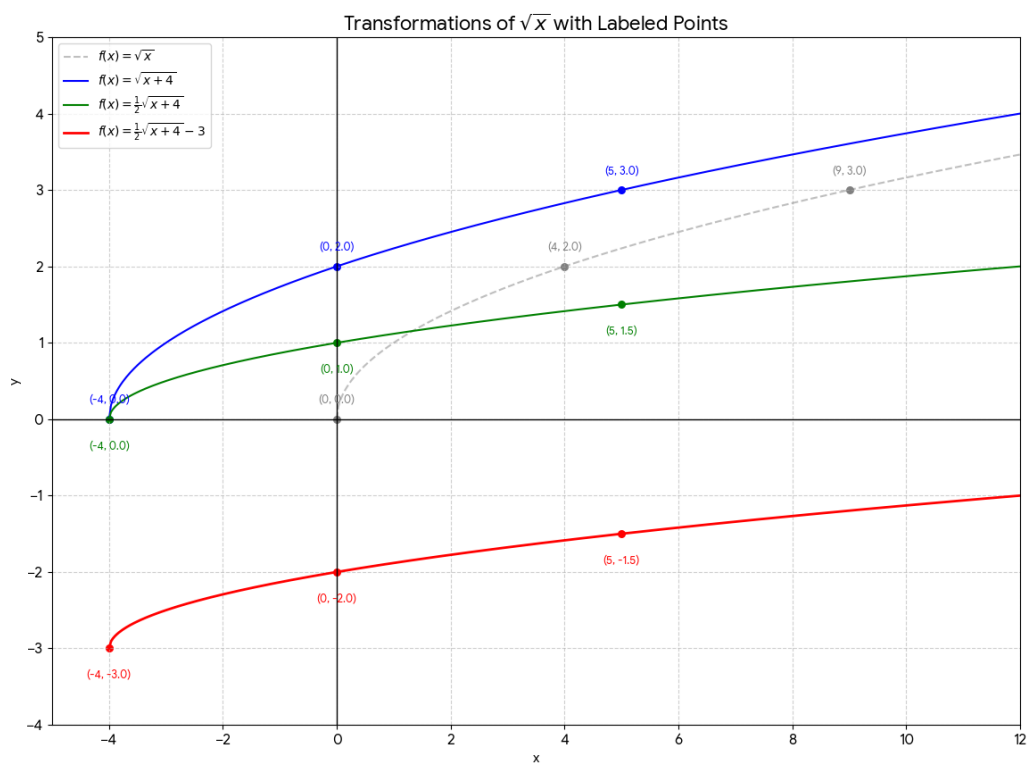
a) $f(x) = \frac{1}{2}\sqrt{x+4} - 3$

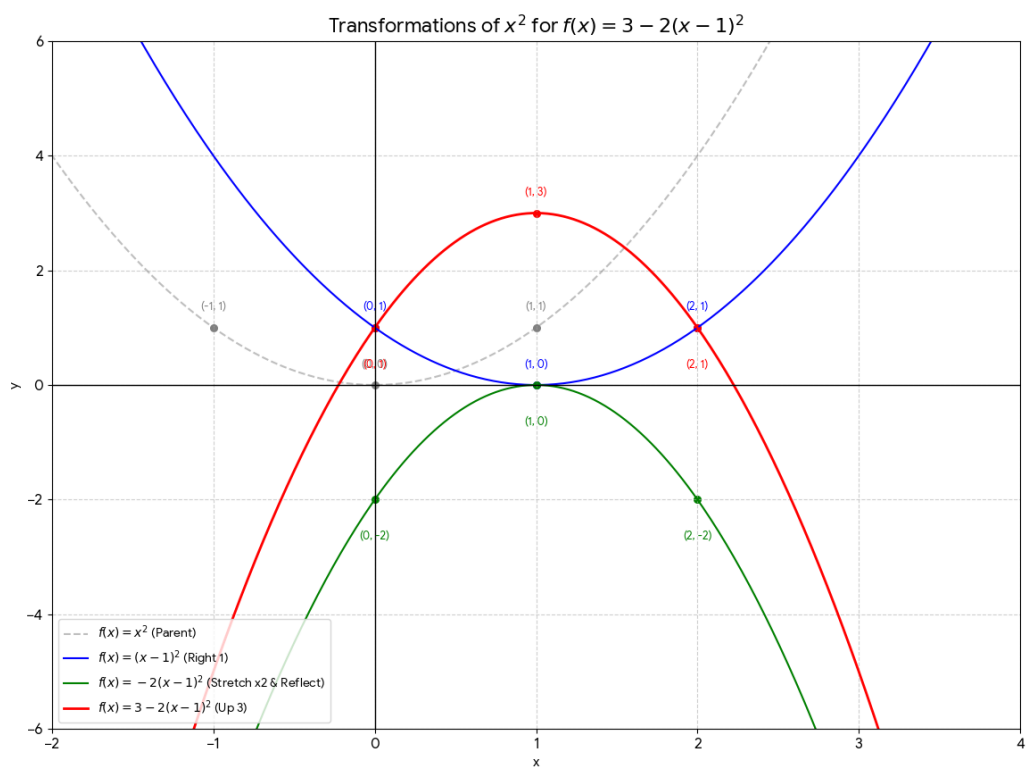
b) $f(x) = 3 - 2(x-1)^2$

Solution: You can try plugging in points, say to the first equation $f(x) = \frac{1}{2}\sqrt{x+4} - 3$ and get things like $f(0) = -1$, $f(1) = \frac{1}{2}\sqrt{5} - 3$, $f(2) = \frac{1}{2}\sqrt{6} - 3$ but these points are clearly not rational and hard to graph without a calculator. The correct approach is to interpret these as a series of transformations on their parent functions.

We start with $f(x) = \sqrt{x}$. We shift to the left by 5, vertically shrink by a factor, and shift down by 3 units.

We shift to the left by 1, stretch by a factor of 2 and reflect it across the x -axis, and shift up by 3





3. A function f is given and the indicated transformations are applied to the graph (in the given order). Write an equation for the final transformed graph.

a) $f(x) = |x|$; shift 2 units to the left and shift downward 5 units.

b) $f(x) = \sqrt[4]{x}$; reflect about the y -axis and shift upward 1 unit.

c) $f(x) = x^2$; shift 2 units to the left and reflect about the x -axis.

d) $f(x) = |x|$; shrink vertically by a factor of $\frac{1}{2}$, shift 1 unit to the left, and shift upward 3 units.

e) $f(x) = x^3 - x$; horizontally stretch by a factor of 2, shift to the left by 1, and shift up by 1.

Solution:

a)

$$|x| \rightarrow |x + 2| \rightarrow |x + 2| - 5$$

b)

$$\sqrt[4]{x} \rightarrow \sqrt[4]{-x} \rightarrow \sqrt[4]{-x} + 1$$

c)

$$x^2 \rightarrow (x + 2)^2 \rightarrow -(x + 2)^2$$

d)

$$|x| \rightarrow \frac{1}{2}|x| \rightarrow \frac{1}{2}|x - 1| \rightarrow \frac{1}{2}|x - 1| + 3$$

e) This is just 1d).

$$f(x) \rightarrow f\left(\frac{1}{2}x\right) \rightarrow f\left(\frac{1}{2}(x + 1)\right) \rightarrow 1 + f\left(\frac{1}{2}(x + 1)\right)$$

Try getting Gemini to visualize this.

Friday 2/27 (Week 5) §2.7 – 2.8

This week, we covered:

- Addition, subtraction, multiplication, and division of functions.
- Composing functions.
- Finding domains of composed functions.
- One-to-one functions and the horizontal line test.
- Algebraically finding the inverse of a function.
- Graphically finding the inverse of a function by reflecting across the line $y = x$.

1. Given the functions

$$f(x) = 1 + x^2 \quad g(x) = \sqrt{x-1}$$

Find the following.

- a) $f(g(x))$
- b) $f(g(2))$
- c) $g(f(x))$
- d) $f(f(2))$
- e) $f(g(f(x)))$
- f) $g(f(g(x)))$

Solution:

- a) $f(g(x)) = 1 + \sqrt{x-1}^2 = 1 + x - 1 = x$
- b) $f(g(2)) = 2$
- c) $g(f(x)) = \sqrt{1+x^2-1} = x$ What does part a) and this say about the two functions f and g ?
- d) $f(f(x)) = 1 + (1+x^2)^2 \implies f(f(2)) = 1 + (1+2^2)^2 = 1 + 5^2 = 26$
- e) $f(g(f(x))) = f(g(1+x^2)) = f(\sqrt{x^2}) = f(x) = 1+x^2$

2. Given the functions

$$f(x) = \sqrt{1-x} \quad g(x) = 1-x^2 \quad h(x) = 1+\sqrt{x}$$

Find $f(g(h(x)))$ and its domain.

Solution:

$$f(g(h(x))) = f(g(1+\sqrt{x})) = f(1-(1+\sqrt{x})^2) = \sqrt{1-(1-(1+\sqrt{x})^2)} = \sqrt{1-1+(1+\sqrt{x})^2} = 1+\sqrt{x}$$

Domain is $[0, \infty)$.

3. Find the inverses of the following functions. Check that they are inverses of each other.

- a) $f(x) = 3x + 15$
- b) $f(x) = 5 - 4x^3$
- c) $f(x) = \frac{2x+5}{x-7}$
- d) $f(x) = x^{3/2} + 1$
- e) $f(x) = (x-2)^{3/5}$

Solution:

a)

$$y = 3x + 15 \implies 3x = y - 15 \implies x = \frac{1}{3}y - 5$$

So $f^{-1}(x) = \frac{1}{3}x - 5$.

Check $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.

$$f(f^{-1}(x)) = f\left(\frac{1}{3}x - 5\right) = 3\left(\frac{1}{3}x - 5\right) + 15 = x$$

Other side is similar.

b) $f(x) = 5 - 4x^3$

$$y = 5 - 4x^3 \implies x^3 = -\frac{1}{4}(y - 5) \implies x = \sqrt[3]{-\frac{1}{4}(y - 5)}$$

So $f^{-1}(x) = \sqrt[3]{-\frac{1}{4}(x - 5)}$

c) $f(x) = \frac{2x+5}{x-7}$

$$y = \frac{2x+5}{x-7} \implies yx - 7y = 2x + 5 \implies yx - 2x = 5 + 7y \implies x = \frac{5+7y}{y-2}$$

So $f^{-1}(x) = \frac{5+7x}{x-2}$

d) $f(x) = x^{3/2} + 1$

$$y = x^{3/2} + 1 \implies y - 1 = x^{3/2} \implies (y - 1)^{2/3} = x$$

So $f^{-1}(x) = (x - 1)^{2/3}$

e) $f(x) = (x - 2)^{3/5}$

$$y = (x - 2)^{3/5} \implies y^{5/3} + 2 = x$$

So $f^{-1}(x) = x^{5/3} + 2$

Friday 3/13 (Week 7) §3.6 – 3.7

This week we covered:

- Rational Functions
 - Vertical, Horizontal, and Slant Asymptotes
 - Removable Discontinuities (Holes)
 - Sketching Rational Functions
- Solving Polynomial and Rational Inequalities.

1. Find all asymptotes, removable discontinuities (holes) and the x and y intercepts.

- a) $\frac{x^2 + 2}{x - 1}$
b) $\frac{5x^3}{x^3 + 2x^2 + 5x}$
c) $\frac{x^2 - 2x - 8}{x}$

Solution:

- a) There is a vertical asymptote at $x = 1$. There is no horizontal asymptote since the degree of the numerator is greater than the degree of the denominator. There is a slant asymptote. Do long division.

$$\begin{array}{r} X + 1 \\ X - 1 \overline{) X^2 + 2} \\ \underline{- X^2 + X} \\ X + 2 \\ \underline{- X + 1} \\ 3 \end{array}$$

So the result is

$$\frac{x^2 + 2}{x - 1} = x + 1 + \frac{3}{x - 1}$$

where $y = x + 1$ is a slant asymptote. There are no x intercepts, and the y intercept is given by $(0, -2)$.

- b) We need to factor the numerator:

$$x^3 + 2x^2 + 5x = x(x^2 + 2x + 5) = x(x^2 + 2x + 1 + 4) = x((x + 1)^2 + 4)$$

So the second factor is a parabola that is always positive. $x = 0$ is a removable discontinuity, so there is no vertical asymptote. Since the degree of the numerator and the degree of the denominator are the same, the horizontal asymptote is $y = 5$. There is no x intercept since there is a hole at $(0, 0)$. Same idea with no y -intercept.

2. Find the intercepts and asymptotes, and sketch a graph of the rational function. State the domain.

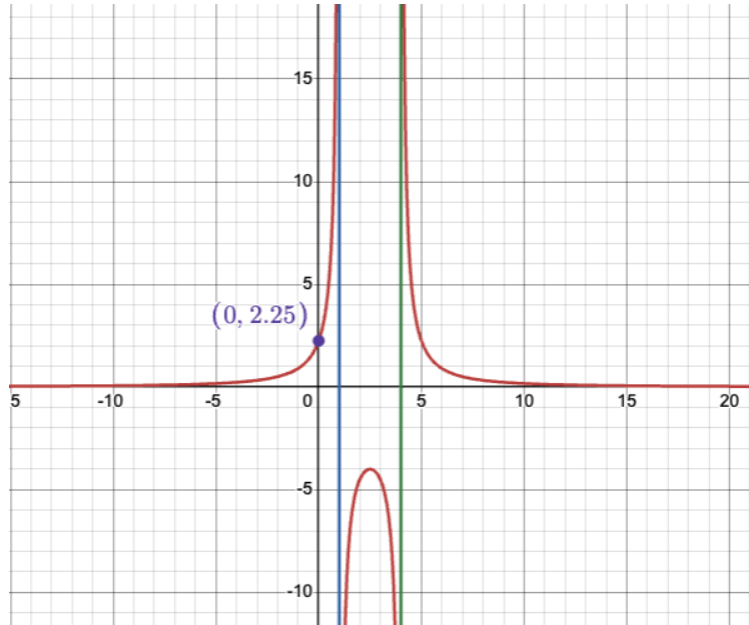
- a) $\frac{9}{x^2 - 5x + 4}$
b) $\frac{x^3 - x^2}{x^3 - 3x - 2}$
c) $\frac{x^3 - 2x^2 - 3x}{x - 3}$

Solution:

- a) Factor the denominator

$$\frac{9}{x^2 - 5x + 4} = \frac{9}{(x-1)(x-4)}$$

Since the degree of the denominator is greater than the degree of the numerator, we have a horizontal asymptote at $y = 0$. We have vertical asymptotes at $x = 1, 4$. This function has no x -intercept and has a y -intercept at $(0, \frac{9}{4})$.



- b) Factor the numerator

$$x^3 - x^2 = x^2(x-1)$$

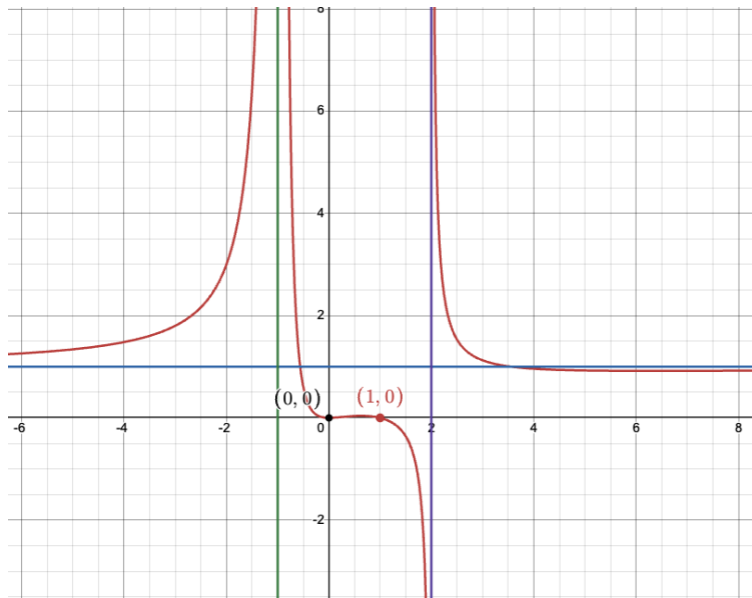
Factor the denominator. By the rational roots theorem, we can tell that $x = -1$ is a root.

$$\begin{array}{r} X^2 - X - 2 \\ X + 1 \overline{) X^3 - 3X - 2} \\ \underline{-X^3 - X^2} \\ -X^2 - 3X - 2 \\ X^2 + X \\ \underline{ -2X - 2} \\ \phantom{ 2X} + 2 \\ \phantom{ 2X} \underline{ 2X + 2} \\ \phantom{\phantom{ 2X} 0} \end{array}$$

The result can be easily factored so we get our original expression is

$$\frac{x^3 - x^2}{x^3 - 3x - 2} = \frac{x^2(x-1)}{(x-2)(x+1)^2}$$

We have vertical asymptotes at $x = -1, 2$ and a horizontal asymptote at $y = 1$. The x -intercepts are $(0, 0), (1, 0)$ and the y -intercept is $(0, 0)$



c)

$$\frac{x^3 - 2x^2 - 3x}{x - 3} = \frac{x(x - 3)(x + 1)}{x - 3} = x(x + 1)^2$$

So this is a polynomial function with a hole at $x = 3$. Graph omitted.

3. Solve the following inequalities.

a) $x^4 + 3x^3 - 3x^2 + 3x - 4 < 0$

b) $\frac{x + 4}{x - 5} \leq 4$

c) $\frac{1}{x + 1} - \frac{1}{x + 2} \leq \frac{1}{(x + 2)^2}$

d) $2x^4 - x^2 - 6 \leq 0$

Solution:

a) $x = 1$ is clearly a zero.

$$\begin{array}{r} X^3 + 4X^2 + X + 4 \\ X - 1 \overline{) X^4 + 3X^3 - 3X^2 + 3X - 4} \\ \underline{-X^4 + X^3} \\ 4X^3 - 3X^2 \\ \underline{-4X^3 + 4X^2} \\ X^2 + 3X \\ \underline{-X^2 + X} \\ 4X - 4 \\ \underline{-4X + 4} \\ 0 \end{array}$$

We can factor the result by grouping.

$$x^4 + 3x^3 - 3x^2 + 3x - 4 = (x - 1)(x^2(x + 4) + x + 4) = (x - 1)(x^2 + 1)(x + 4)$$

Set up the sign chart at $x = 1, -4$ to get that the solution is $(-4, 1)$.

b) Move everything to one side.

$$\frac{x + 4}{x - 5} \leq 4 \implies \frac{x + 4 - 4(x - 5)}{x - 5} \leq 0 \implies \frac{-3x + 24}{x - 5} \leq 0$$

Create the sign chart with $x = 5, 8$ to conclude the solution is $(-\infty, 5) \cup [8, \infty)$. We don't include 5 to avoid division by 0.

c) Move everything to one side and combine.

$$\begin{aligned} \frac{1}{x + 1} - \frac{1}{x + 2} &\leq \frac{1}{(x + 2)^2} \implies \frac{1}{x + 1} - \frac{1}{x + 2} - \frac{1}{(x + 2)^2} \leq 0 \\ &\implies \frac{(x + 2)^2}{(x + 1)(x + 2)^2} - \frac{(x + 1)(x + 2)}{(x + 1)(x + 2)^2} - \frac{x + 1}{(x + 2)^2} \leq 0 \\ &\implies \frac{x^2 + 4x + 4 - x^2 - 3x - 2 - x - 1}{(x + 1)(x + 2)^2} \leq 0 \\ &\implies \frac{1}{(x + 1)(x + 2)^2} \leq 0 \end{aligned}$$

Create the sign chart with $x = -2, -1$ to conclude $(-\infty, -2) \cup (-2, -1)$.

d) Directly factor

$$\begin{aligned}2x^4 - x^2 - 6 \leq 0 &\implies (2x^2 + 3)(x^2 - 2) \leq 0 \\ &\implies (2x^2 + 3)(x - \sqrt{2})(x + \sqrt{2}) \leq 0\end{aligned}$$

Create the sign chart at $x = -\sqrt{2}, \sqrt{2}$ to conclude

$$[-\sqrt{2}, \sqrt{2}]$$

Friday 3/20 (Week 8) §4.1 – 4.4

This week we covered:

- Exponential functions $f(x) = a^x$ and its graph.
- The concept of e (but not covered very deeply)
- Logarithmic functions $f(x) = \log_a x$
- The concept of the natural log $\log_e x = \ln x$.
- Evaluating simple logarithms.
- Properties of logarithms.
- Plotting logarithms.
- Finding domains of logarithmic functions.

1. Graph the function not by plotting points but by applying transformations to a parent function. State the x and y -intercepts, domain, range, and horizontal asymptote.

a) $f(x) = 2^{x-4} + 1$

b) $f(x) = 1 - 3^{-x}$

c) $f(x) = -e^{x-1} - 2$.

Solution: Done in class. See Wolfram Alpha/Desmos.

2. Graph the function not by plotting points but by applying transformations to a parent function. State the $x - y$ -intercept, domain, range, and horizontal asymptote.

a) $f(x) = 2 - \log_{1/3} x$

b) $f(x) = \log |x|$

Solution: Done in class. See Wolfram Alpha/Desmos.

3. Find the domains of the following functions.

a) $\log_3(x^2 - 1)$

b) $\sqrt{x - 2} - \log_5(10 - x)$

Solution:

a) The argument of the logarithm must be positive. Solving the inequality $x^2 - 1 > 0$ gives us $(-\infty, -1) \cup (1, \infty)$

b) We require $x \geq 2$ and $10 - x > 0 \implies x < 10$. Combined this gives us $[2, 10)$.

4. Evaluate the following expressions.

a) $e^{\ln 10}$

b) $\log_8 0.25$

c) $\log_4 \sqrt{2}$

d) $\log_{1/3} 27$

e) $\log_9 \sqrt{3}$

f) $-\frac{1}{3} \log_3 27$

g) $\log_2 6 - \log_2 15 + \log_2 20$

- h) $\log_4 16^{100}$
 i) $\log(\log 10^{10000})$

Solution:

- a) $e^{\ln 10} = 10$
 b) $\log_8 0.25 = \log_8 \left(\frac{1}{4}\right) = \log_{2^3} 2^{-2} = -\frac{2}{3}$
 c) $\log_4 \sqrt{2} = \frac{1}{4}$
 d) $\log_{1/3} 27 = -3$
 e) $-\frac{1}{3} \log_3 27 = -\frac{1}{3}(3) = -1$
 f) $\log_2 6 - \log_2 15 + \log_2 20 = \log_2 8 = 3$
 g) $\log_4 16^{100} = 100 \log_4 16 = 100(2) = 200$
 h) $\log(\log 10^{10000}) = \log(10000 \log 10) = \log(10000) = 4$

5. Use the laws of logarithms to expand the expression. Bring down all exponents.

- a) $\log_8 \left(\frac{a^3 b^2}{c}\right)$
 b) $\log \sqrt[3]{x^2 + 4}$
 c) $\ln \sqrt{\frac{x^2 + y^2}{x + y}}$
 d) $\ln \left(\frac{x^2}{\sqrt{x+1}}\right)$

Solution:

a)

$$\log_8 \left(\frac{a^3 b^2}{c}\right) = \log_8 a^3 + \log_8 b^2 - \log_8 c = 3 \log_8 a + 2 \log_8 b - \log_8 c$$

b)

$$\log \sqrt[3]{x^2 + 4} = \frac{1}{3} \log \sqrt{x^2 + 4}$$

c)

$$\ln \sqrt{\frac{x^2 + y^2}{x + y}} = \frac{1}{2} \ln \frac{x^2 + y^2}{x + y} = \frac{1}{2} (\ln(x^2 + y^2) - \ln(x + y))$$

d)

$$\ln \left(\frac{x^2}{\sqrt{x+1}}\right) = \ln x^2 - \ln \sqrt{x+1} = 2 \ln x - \frac{1}{2} \ln(x+1)$$

6. Use the laws of logarithms to combine the expressions.

- a) $\log(x+1) + \log(x-1) - 3 \log x$
 b) $\frac{1}{2} [\ln(x-4) + 5 \ln(x^2 + 4x)]$

Solution:

a)

$$\log(x+1) + \log(x-1) - 3 \log x = \log(x+1) + \log(x-1) - \log x^3 = \log \left(\frac{x^2 - 1}{x^3}\right)$$

b)

$$\frac{1}{2} [\ln(x-4) + 5 \ln(x^2 + 4x)] = \frac{1}{2} \ln((x-4)(x^2 + 4x)^5) = \ln(\sqrt{(x-4)(x^2 + 4x)^5})$$

Friday 3/27 (Week 9) §4.5, §5.1

This week we covered:

- Solving logarithmic and exponential equations.
- The unit circle.
- Terminal points on the unit circle

1. Solve the following equations.

a) $e^{3x-2} = e^{x^2}$

b) $5\left(\frac{2}{3}\right)^{3x+1} = 6$

c) $10^{x+3} = 6^{2x}$

d) $5 \ln(3-x) = 4$

e) $\log(x+1) + \log 2 = \log(5x)$

f) $\log_4(x+3) + \log_4(x-1) = 2$

Solution:

a) Take natural logs of both sides and solve the quadratic

$$3x - 2 = x^2 \implies x^2 - 3x + 2 = 0 \implies (x-1)(x-2) = 0 \implies x = 1, 2$$

b)

$$\begin{aligned} 5\left(\frac{2}{3}\right)^{3x+1} = 6 &\implies \left(\frac{2}{3}\right)^{3x+1} = \frac{6}{5} \\ &\implies (3x+1) \ln\left(\frac{2}{3}\right) = \ln\left(\frac{6}{5}\right) \\ &\implies 3x+1 = \frac{\ln\left(\frac{6}{5}\right)}{\ln\left(\frac{2}{3}\right)} \\ &\implies x = \frac{1}{3} \left[\frac{\ln\left(\frac{6}{5}\right)}{\ln\left(\frac{2}{3}\right)} - 1 \right] \end{aligned}$$

c)

$$\begin{aligned} 10^{x+3} = 6^{2x} &\implies (x+3) \log 10 = 2x \log 6 \\ &\implies 2x \log 6 - (\log 10)x = 3 \log 10 \\ &\implies x = \frac{3 \log 10}{2 \log 6 - \log 10} \end{aligned}$$

d)

$$5 \ln(3-x) = 4 \implies \ln(3-x) = \frac{4}{5} \implies e^{4/5} = 3-x \implies x = 3 - e^{4/5}$$

e)

$$\begin{aligned} \log(x+1) + \log 2 = \log(5x) &\implies \log\left(\frac{2(x+1)}{5x}\right) = 0 \\ &\implies \frac{2(x+1)}{5x} = 1 \\ &\implies 2x+2 = 5x \\ &\implies x = \frac{2}{3} \end{aligned}$$

f)

$$\begin{aligned}\log_4(x+3) + \log_4(x-1) = 2 &\implies \log_4((x+3)(x-1)) = 2 \\ &\implies (x+3)(x-1) = 16 \\ &\implies x^2 + 2x - 3 = 16 \\ &\implies x^2 + 2x + 1 = 20 \\ &\implies (x+1)^2 = 20 \\ &\implies x = -1 \pm \sqrt{20}\end{aligned}$$

The negative one is an extraneous solution, so $x = -1 + \sqrt{20}$.

2. Show the points lie on the unit circle.

a) $\left(-\frac{24}{25}, -\frac{7}{25}\right)$

b) $\left(-\frac{5}{7}, -\frac{2\sqrt{6}}{7}\right)$

Solution:

a) $\left(-\frac{24}{25}\right)^2 + \left(-\frac{7}{25}\right)^2 = \frac{576+49}{625} = \frac{625}{625} = 1$

b) $\left(-\frac{5}{7}\right)^2 + \left(-\frac{2\sqrt{6}}{7}\right)^2 = \frac{25}{49} + \frac{24}{49} = 1$

3. Find the requested point on the unit circle.

a) The x coordinate is $\frac{5}{13}$ and the y -coordinate is negative.

b) The y coordinate is $-\frac{2}{7}$ and the point is in quadrant IV.

Solution:

a) $\left(\frac{5}{13}\right)^2 + y^2 = 1 \implies y^2 = \frac{144}{169} \implies y = -\frac{12}{13}$.

b) The x -coordinate must be positive. So

$$x^2 + \left(-\frac{2}{7}\right)^2 = 1 \implies x^2 = \frac{45}{49} \implies x = \frac{3\sqrt{5}}{7}$$

4. Find the terminal point on the unit circle for each value of t

a) $t = \frac{11\pi}{6}$

b) $t = \frac{13\pi}{4}$

c) $t = \frac{41\pi}{6}$

d) $t = -\frac{11\pi}{3}$

e) $t = -\frac{41\pi}{4}$

Solution:

a) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

b) Subtract off $2\pi = \frac{8\pi}{4}$ to get $t = \frac{5\pi}{4} \implies \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

c) Subtract off $2\pi = \frac{12\pi}{6}$ to get $t = \frac{29\pi}{6}$. Do it again to get $t = \frac{13\pi}{6}$. One more time to get $t = \frac{\pi}{6}$.

So $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

d) Add $2\pi = \frac{6\pi}{3}$ to get $t = -\frac{5\pi}{3}$. One more time to get $t = \frac{\pi}{3}$. So $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

e) This is -10.25π . So I need to add $11\pi = \frac{44\pi}{4}$ to get $t = \frac{3\pi}{4}$. So $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

Friday 4/03 (Week 10) §5.2 – 5.3

This week we covered:

- Evaluating \sin , \cos , \tan , \csc , \sec , \cot at integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$.
- Evenness and oddness of the six trig functions.
- Graphing \sin and \cos and its transformations.
- Amplitudes and periods of \sin and \cos .

1. Evaluate the following.

- $\sin\left(\frac{4\pi}{3}\right)$
- $\sec\left(\frac{11\pi}{6}\right)$
- $\cot\left(-\frac{\pi}{3}\right)$
- $\csc\left(\frac{2\pi}{3}\right)$
- $\sec\left(-\frac{5\pi}{3}\right)$
- $\cos\left(\frac{10\pi}{3}\right)$
- $\sin(13\pi)$
- $\cos(14\pi)$
- $\tan(15\pi)$
- $\sin\left(\frac{25\pi}{2}\right)$
- $\cos\left(\frac{25\pi}{2}\right)$
- $\tan\left(\frac{25\pi}{2}\right)$

Solution:

- $\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$
- $\sec\left(\frac{11\pi}{6}\right) = \frac{1}{\cos\left(\frac{11\pi}{6}\right)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$
- $\cot\left(-\frac{\pi}{3}\right) = \cot\left(\frac{5\pi}{3}\right) = \frac{\cos\left(\frac{5\pi}{3}\right)}{\sin\left(\frac{5\pi}{3}\right)} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$
- $\csc\left(\frac{2\pi}{3}\right) = \frac{1}{\sin\left(\frac{2\pi}{3}\right)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$
- $\sec\left(-\frac{5\pi}{3}\right) = \sec\left(\frac{\pi}{3}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)} = \frac{1}{\frac{1}{2}} = 2$
- $\cos\left(\frac{10\pi}{3}\right) = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$
- $\sin(13\pi) = \sin(\pi) = 0$
- $\cos(14\pi) = \cos(0) = 1$
- $\tan(15\pi) = \tan(\pi) = \frac{\sin(\pi)}{\cos(\pi)} = \frac{0}{-1} = 0$
- $\sin\left(\frac{25\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$
- $\cos\left(\frac{25\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$
- $\tan\left(\frac{25\pi}{2}\right) = \tan\left(\frac{\pi}{2}\right) = \frac{1}{0}$ so undefined

2. Determine whether the following are even or odd or neither.

- $f(x) = x^2 \sin x$
- $f(x) = \sin x \cos x$
- $f(x) = x^3 + \cos x$

Solution: Recall that sin is odd so $\sin(-x) = -\sin x$. cos is even so $\cos(-x) = \cos x$.

a) $f(-x) = x^2 \sin(-x) = -x^2 \sin x = -f(x)$ so odd.

b) $f(-x) = \sin(-x) \cos(-x) = -\sin x \cos x = -f(x)$ so odd

c) $f(-x) = -x^3 + \cos(-x) = -x^3 + \cos x$ so neither.

3. Graph one period of the following. Determine the length of the period and the amplitude.

a) $y = 2 \sin\left(\frac{2}{3}x - \frac{\pi}{6}\right)$

b) $y = 5 \cos\left(3x - \frac{\pi}{4}\right)$

c) $y = 2 - 2 \cos\left(3\left(x + \frac{\pi}{3}\right)\right)$

d) $y = 2 + \sin\left(2\pi\left(x + \frac{1}{8}\right)\right)$

Solution:

Friday 4/24 (Week 12) §6.1, §7.1 – 7.2

This week we covered:

- ???

1. Test Problem

Solution:

Friday 5/08 (Week 14) §9.1 – 9.2

No discussion problems - this is mere days before the final so open it up for questions (office hours).