Qualifying Exam Syllabus

Andrew Shi

Date: Monday 4/8/2019 Time: 2-5pm Location: 891 Evans

Committee: L. Craig Evans, Per-Olof Persson (Advisor), John Strain (Chair), Shawn Shadden (Mechanical Engineering)

Major Topic: Numerical Linear Algebra (Applied Mathematics)

- **Linear Least Squares:** Singular Value Decomposition, QR Factorization, Gram-Schmidt orthogonalization, Householder reflections, Givens rotations, Normal equations (Trefethen I and II)
- **Linear Equations:** Norms, condition number, backwards stability, LU Factorization, Cholesky factorization (Trefethen III and IV)
- **Eigenvalue Problems:** Reduction to Hessenberg and tridiagonal form, Rayleigh quotient, power iteration, inverse iteration, QR algorithm. (Trefethen V)
- **Iterative Methods:** Stationary iterative methods (Jacobi, Gauss-Seidel, SOR), Arnoldi and Lanczos iteration, GMRES, Conjugate Gradient method (Trefethen VI)

References: Trefethen and Bau, *Numerical Linear Algebra*. Persson Math 221 Notes.

Major Topic: Numerical Solution to Differential Equations (Applied Mathematics)

- **Basic Theory:** consistency, stability, convergence, stiff problems, absolute stability, A-stability, $A(\alpha)$-stability, L-stability.
- **Runge-Kutta Methods:** Butcher tableau, order conditions, DIRK, fully-implicit RK methods, error estimation and stepsize control
- **Finite Difference Methods:** consistency, stability, convergence, Fourier stability analysis, CFL condition, method of lines and basic schemes (Crank-Nicolson, leapfrog, Lax-Friedrichs, Lax-Wendroff, upwinding)
- **Hyperbolic Conservation Laws**: Shocks, rarefaction fans, Rankine-Hugoniot condition, entropy conditions, Riemann problem, Godunov's method

- **Finite Element Methods:** Weak form, piecewise polynomial bases, high-order, quadrature rules, isoparametric elements

References: LeVeque, *Finite Difference Methods for Ordinary and Partial Differential Equations.* LeVeque, *Finite Volume Methods for Hyperbolic Problems.* Hughes, *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis.* Persson Math 228ab Notes.

Minor Topic: Partial Differential Equations (Mathematical Analysis)

- **Laplace's Equation:** fundamental solution, mean-value formulae, properties of harmonic functions, Green's function, energy methods (2.2)
- **Heat Equation:** fundamental solution, mean-value property, maximum principle, uniqueness, regularity, energy methods (2.3)
- **Method of Characteristics:** derivation, boundary conditions, local solution (3.2)
- **Introduction to Hamilton Jacobi Equations:** Euler-Lagrange Equation, Hamilton's ODE, Legendre Transformation, Hopf-Lax Formula, Weak Solutions (3.3)
- **Scalar Conservation Laws:** Shocks, entropy condition, Rankine-Hugoniot condition, Lax-Oleinik formula, weak solutions, Riemann's problem (3.4 except 3.4.5)
- **Sobolev Spaces:** definition, weak solutions, approximation by smooth functions, extensions, traces, Gagliardo-Nirenberg-Sobolev inequality (5.1-5.6)

References: Evans, *Partial Differential Equations*