1. [10 points] The figure below shows the level curves (which correspond to values spaced at intervals of 0.25) of a function $f(x, y)$. (The labels on the curves indicate the values of the function. Be sure to note the minus signs on some of them.)

(a) Describe the region in the $(x, y)$ plane on which the partial derivatives $f_x(x, y)$ and $f_y(x, y)$ are both negative. Explain how you arrived at your answer.
(b) Find, as accurately as you can from the figure, the line integral of the gradient vector field $\nabla f$ along the segment of the $y$ axis which begins at the origin and ends at the top of the figure.
(c) The level curves are furthest apart near the center of the figure. What does this tell you about the gradient of $f$?
2. [10 points] Find the **maximum** value of each of the following functions on the region $4x^2 + y^2 \leq 4$.
   (a) $s(x, y) = 10 - 5x^2 - 9y^2$.
   (b) $t(x, y) = x - y$.

3. [10 points] Evaluate the surface integral
   \[
   \int \int_S \nabla \times \mathbf{F} \cdot d\mathbf{S}
   \]
   of the curl of the vector field $\mathbf{F}(x, y, z) = xyz \mathbf{i} + 2x \mathbf{j} + e^{xy} \cos z \mathbf{k}$, where $S$ is the hemisphere $x^2 + y^2 + z^2 = 1$, $z \leq 0$ with the orientation determined by the “upward” normals.

4. [10 points]
   (a) Use a triple integral to find the volume of the solid region bounded by the surfaces $x = y^2$, $z = 0$, and $x + z = 4$.
   (b) Find the flux of the vector field $x \mathbf{i} - 2y \mathbf{j} + z \mathbf{k}$ through the boundary of the region in part (a), with the outward orientation.
   (c) Find the flux of the vector field $x \mathbf{i} + 2y \mathbf{j} + z \mathbf{k}$ through the boundary of the region in part (a), with the outward orientation.

5. [10 points] Evaluate the following line integral in two ways:
   (a) directly, using a parametrization of the curve;
   (b) with the help of Green’s theorem.
   \[
   \int_C ((x + 2y) \mathbf{i} - (x - 2y) \mathbf{j}) \cdot d\mathbf{r},
   \]
   where $C$ consists of the line segment from $(0, 0)$ to $(1, 1)$, followed by the arc of the parabola $y = x^2$ from $(1, 1)$ to $(0, 0)$.

6. [10 points] Let $D$ be the plane region bounded by the arcs described in polar coordinates by $r = 1$ and $r = 1 + \frac{1}{2} \cos \theta$, both for $\pi/2 \leq \theta \leq 3\pi/2$.
   Sketch the region $D$ and find its area. (Be sure that your answer for the area is consistent with your sketch.)

7. [10 points] Here is some information about a function $f(x, y)$ and its partial derivatives:
   $f(1, 7) = 3$, $f_x(1, 7) = -5$, $f_y(1, 7) = 2$.
   (a) Find the equation of the tangent line to the level curve $f(x, y) = 3$ at the point $(1, 7)$.
   (b) Find the equation of the tangent plane to the graph of $f$ at the point $(1, 7, 3)$.
   (c) Describe the plane in part (b) as a level surface of a function of three variables.

8. [10 points]
   (a) Is there a function $f(x, y, z)$ whose gradient vector field is $y \mathbf{k}$? If so, find one. If not, explain why not.
   (b) Is there a vector field $\mathbf{F}(x, y, z)$ whose curl is $y \mathbf{k}$? If so, find one. If not, explain why not.
   (c) Is there a vector field $\mathbf{F}(x, y, z)$ whose divergence is $y \mathbf{k}$ If so, find one. If not, explain why not.