Problem 1. Solve the linear system
\[
\begin{align*}
x + 3y - 2z &= 0 \\
2x - y + z &= 0 \\
4x + 4y - 3z &= 0
\end{align*}
\]
(10 points). Describe the set of solution on a picture (5 points). What happens to this picture if you change the zeros on the right to any number (5 points)?

Problem 2. True or false?
\begin{itemize}
\item The product of two non-zero matrices is always non-zero.
\item The vectors
\[
\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 7 \\ 4 \\ -1 \end{pmatrix}
\]
are not linearly independent.
\item The linear span of
\[
\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}
\]
is the plane of equation $7x - 6y - 5z = 0$.
\item If $x, y, z$ are three numbers all non-zero and $\vec{u}, \vec{v}, \vec{w}$ are three vectors all non-zero then $x\vec{u} + y\vec{v} + z\vec{w}$ is non-zero.
\end{itemize}

Problem 3 (20 points) Let $T$ be the 90 degree rotation clockwise. Find
\[
T \begin{pmatrix} 1 \\ 2 \end{pmatrix}, T \begin{pmatrix} -1 \\ 1 \end{pmatrix}, T \begin{pmatrix} x \\ y \end{pmatrix}, x, y \text{ are any numbers.}
\]
(10 points). Then find a matrix $A$ such that
\[
T \begin{pmatrix} x \\ y \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix}
\]
(5 points). Find $A^4$ and interpret geometrically (5 points).

Problem 4. (20 points) Let $A$ be the matrix
\[
A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.
\]
Find $A^n$ where $n$ is any number. Hint: first find $A^2, A^3, \ldots$

Problem 5. Find the inverse of the matrix
\[
\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 3 & 5 \end{bmatrix}
\]
(15 points). Double check your answer (5 points).