Qualifying exam syllabus

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Friday, May 13, 2011
959 Evans Hall
10 a.m.

QUALIFYING EXAM COMMITTEE
Prof. Ian Agol
Prof. Denis Auroux (*committee chair*)
Prof. Peter Teichner (*advisor*)
Prof. Alan Nelson (*English department*)

MAJOR TOPIC
Algebraic topology
Geometry and topology

Homology and cohomology. Simplicial/singular/cellular/axiomatic homology and cohomology, long exact sequences of a pair, Mayer-Vietoris sequences, excision, Lefschetz fixed point theorem, universal coefficient theorems, Künneth formula, cup and cap products, Poincaré duality, classifying spaces, cohomology operations, Serre spectral sequence.

Homotopy theory. Fundamental group, van Kampen's theorem, covering spaces, higher homotopy groups, cellular approximation, CW approximation, Whitehead's theorem, Freudenthal suspension theorem, stable homotopy groups, Hurewicz theorem, long exact sequence of a fibration, Postnikov towers.

K-theory. Bott periodicity, six-term exact sequence, characteristic classes, cobordism, Thom spaces, Pontrjagin-Thom construction.

References: Hatcher, *Algebraic Topology*. Hatcher, *Vector Bundles and K-Theory*. Milnor & Stasheff, *Characteristic Classes*. Switzer, *Algebraic Topology – Homology and Homotopy*.

MAJOR TOPIC
Riemannian geometry
Geometry and topology

Smooth manifolds. Morphisms, orientation, local coordinates, partitions of unity, tangent and cotangent bundles, differential forms.

Riemannian manifolds. Metrics, connections, Levi-Civita theorem, geodesics, convex neighborhoods, Riemannian curvature, sectional curvature, covariant tensors, covariant differentiation of covariant tensors, Jacobi fields, conjugate points, completeness, Hopf-Rinow theorem.

Path spaces. Path spaces and their tangent spaces, variations of paths, energy function, first and second variation formulas, Morse index theorem, Freudenthal suspension theorem, homotopy equivalence of piecewise smooth path space and full path space.

Complex Bott periodicity. Symmetric spaces, compact Lie groups, Morse-Bott functions, complex Bott periodicity.

References: do Carmo, *Riemannian Geometry*. Milnor, *Morse Theory*.

MINOR TOPIC
Complex analysis
Mathematical analysis

Analysis. Cauchy-Riemann equations, Cauchy integral formula, Cauchy-Goursat theorem, Morera's theorem, Liouville's theorem, open mapping theorem, maximum modulus principle, identity theorem, Taylor and Laurent series, residue theorem, contour integrals, Riemann mapping theorem (statement).

Riemann surfaces. Local model, branched coverings, Riemann-Hurwitz formula, sheaves, analytic continuation, differential forms, sheaf cohomology, long exact sequence in sheaf cohomology, Dolbeault's lemma (statement), divisors, line bundles, Riemann-Roch theorem (statement), Serre duality (statement).

References: Brown & Churchill, *Complex Variables and Applications*. Forster, *Lectures on Riemann Surfaces*.