Math 55: Discrete Mathematics
Williams, Spring 2018
GSI: Ai

# Week 9: Recurrence Relations, Generating Functions

Summary:

- This week, the key problem is recurrence relations. Key methods are 1. (memorizing) the special case of constant coefficient linear homogeneous relations, and 2. generating functions.
- This week will also cover the (very general) technique of the inclusion exclusion principle.

1. (Sturmfels Spr09 Final) A new employee checks the hats of six people at the opera, forgetting to put claim check numbers on them. When people come back for their hats, the checker returns hats chosen at random from the remaining has. What is the probability that no one receives his own hat? What is the expected number of hats that are returned correctly?

2. (Sturmfels Spr12 Final) Solve the recurrence relation
$$a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$$
with initial conditions $a_0 = 1, a_1 = 0, a_2 = 7$.

3. (Sturmfels Spr12 Final) Use generating functions to determine the number of different ways 10 identical balloons can be given to four children if each child receives at least two balloons. (Children are distinguishable.)

4. (Ribet Spr13 Final) Find a linear homogeneous recurrence relation of degree 2 with constant coefficients that is satisfied by $a_1 = 1, a_2 = 5, a_3 = 19, a_4 = 65, a_5 = 211, a_6 = 665$. Solve this recurrence relation and compute $a_7, a_8$.