Math 55: Discrete Mathematics
Williams, Spring 2018
GSI: Ai

Week 8: More Probability

- Pick a real number between 0 and 1 uniformly at random. Then pick another number; compare with the previous, and toss the smaller of the two. Repeat ten times (until you have picked 10 numbers in total, tossed 9). What is the probability that in the last step, you toss the newest number?
- Pick ten real numbers between 0 and 1 uniformly at random, and write them in order. What is the probability the last number is largest?

1. (Ribet Spr13) (From last week) Before going on vacation for a week, you ask a forgetful friend to water your plan. Without water, the plant has a 90 percent chance of dying. With watering, it has a 20 percent chance of dying. The probability that your friend will forget to water the plant is 30 percent. What is the probability that your plant will survive the week? If you return to a dead plant, what is the probability your friend forgot to water it?

2. (Ribet Spr13) (From last week) Bob and Alice toss a fair coin repeatedly until either two tails have come up in a row or the sequence heads-tails has come up. Bob wins the first case; Alice wins the second. What is the probability that Alice wins?

3. (Ribet Spr15) Eve has a bag of 12 biased coins. Six of these come up heads 3/5 of the time, while the other six come up tails 2/3 of the time. Eve pulls out a coin at random and tosses it. It comes up heads. What is the probability that she pulled out a coin that is biased toward heads?

4. (Sturmfels Spr12) Mike selects a ball by first picking one of two boxes at random and then selecting a ball from this box at random. The first box contains 4 red balls and 3 blue balls, and the second box contains 2 red balls and 5 blue balls. What is the probability that Mike picked a ball from the second box if he has selected a red ball?

5. (Sturmfels Spr12) A fair die is rolled until the number 6 appears for the first time. After this has happened, no further dice rolls are made. Find the probability that exactly four rolls are made. Also find the expected number of dice rolls.

6. (Sturmfels Spr09) A thumb tack is tossed until it first lands with its point down, at which time no more tosses are made. On each tack toss, the probability of landing point down is 1/3. Find the probability that exactly five tosses are made. Also find the expected number of tosses.

7. (Sturmfels Spr09) Let X_n be the random variable that equals the number of tails minus the number of heads when n fair coins are flipped. Find the expected value and the variance of X_n.