Math 55: Discrete Mathematics
Williams, Spring 2018
GSI: Ai

Week 6: More Induction, Counting

1. (Ribet Spr15) For each $n \geq 0$, let c_n be the number of ways of writing n as a sum of 1's and 2's, where different orders count as different ways. For instance, the ways to write 4 are

$$1+1+1+1, \quad 2+1+1, \quad 1+2+1, \quad 1+1+2, \quad 2+2,$$

so $c_4 = 5$. Use strong induction to prove that c_n is the Fibonacci number f_{n+1}.

2. (Sturmfels Spr12) At a certain university, each student comes from precisely one of the 50 states. What is the minimum number of students who must be enrolled to guarantee that there are at least 100 who come from the same state?

3. (Sturmfels Spr12) Consider the three integers $34^{1000} - 7^{2342} + 3^{177}, 19^{1212} - 9^{2399} + 2^{2001}, 24^{2342} - 5^{8984} + 7^{1999}$. Prove that the product of two of these numbers is nonnegative.

4. (Sturmfels Spr09) Consider all permutations of the letters ABCDEFG. a) How many of these permutations contain strings ABC and DE (each as a consecutive substring)? b) How many permutations has A preceding B, not necessarily immediately?

5. (Ribet Spr15) Let $p \geq 7$ be a prime number. Show that the $(p-1)$-digit number 11...11 is a multiple of p. (GSI Hint: Show that p divides 99...99. Another hint: Fermat's Little Theorem.)