Math 55: Discrete Mathematics
Williams, Spring 2018
GSI: Ai

Week 4: Number Theory, Primes, Divisibility, Inverses

Warm up questions:

- Make sure you know the Euclidean algorithm for finding GCD, and for finding the candidates for Bezout's Theorem.
- Use the Fundamental Theorem of Arithmetic to prove Lemma 1 from Lecture 7: If a, b, c are positive integers with $\gcd(a, b) = 1$ and $a|bc$, then $a|c$. In fact, the Fundamental Theorem of Arithmetic is a good (though overpowered) way to think about many notions regarding GCD, LCM.

All of the following should be viable midterm questions (though not necessarily easy, in my opinion).

1. (Ribet Spr13) Find the smallest positive multiple of 100 that leaves remainder 9 when divided by 19.

2. (Ribet Spr15) Using the identity $1 = 54 \cdot 129 - 35 \cdot 199$, write down an integer that is congruent to 35 (mod 129) and to 54 (mod 199). You can leave your answer as an arithmetic expression (i.e. one involving products, sums, differences).

3. (Sturmfels Spr12) What amount of postage can be formed using only 5-cent and 6-cent stamps? Formulate a conjecture and prove it. (GSI commentary: This isn't so easy so don't waste too much time on it. Make sure you understand why it's not so easy, though.)

4. (Sturmfels Spr12) Determine an integer n such that
$$n \equiv 1 \pmod 7, \quad n \equiv 3 \pmod 8, \quad n \equiv 2 \pmod 9.$$

5. (Sturmfels Spr09) Find an inverse of 81 modulo 250.