Math 55: Discrete Mathematics
Williams, Spring 2018
GSI: Ai

Week 3: Cardinality and Numbers

Warm up questions:

- Group the following into finite, countably infinite, uncountable: $\{0,1,2\}$, the positive integers, the odd integers, the integers, the rational numbers, the real numbers, the complex numbers, the interval $[0,1]$, the set of function from a given finite domain to a given finite codomain.
- What about the set of functions from a given countably infinite domain to a given finite codomain? (Hint: there are two possibilities, depending on the given objects. Also, one of the below exercises is relevant.) What about the set of functions from a given finite domain to a given countably infinite domain?

1. (Ribet Spr15) Let $\{a_n\}$ be the sequence of positive odd numbers defined by: $a_0 = 3$ and $a_n = a_0 a_1 ... a_{n-1} + 2$ for $n \geq 1$. The sequence begins $3, 5, 17, 257, 65537, ...$ If n and m are natural numbers with $m < n$, show that 1 is the only positive integer that divides both a_m and a_n.

2. (Ribet Spr13) Consider the set of all sequences $\{a_n\}$ whose terms a_n are binary digits (in other words, each a_n is 0 or 1). Show that this set is uncountable.

3. (Ribet Spr13) Suppose that p is a prime number and that x and y are integers. Show that if xy and $x+y$ are both divisible by p, then each of x and y is divisible by p.

4. (Sturmfels Spr12) Which amounts of postage can be formed using only 5-cent and 6-cent stamps? Formualte a conjecture and prove it.

5. (Sturmfels Spr12) Compute the following remainders:
   - $19^{145} \mod 13$
   - $(-12)^{36} \cdot 50^{19} \mod 7$.

6. (Sturmfels Spr12) Give an example of two uncountable sets A and B such that the intersection $A \cap B$ is (a) finite, (b) countable infinite, or (c) uncountable.