Math 55: Discrete Mathematics
Williams, Spring 2018
GSI: Ai

Week 2: Proofs and Sets

Warm up questions:

- List all the proof strategies you can think of.
- Proposition, Lemma, Corollary, Conjecture: When would you use each of these words, instead of "Theorem"?
- Write precisely (in propositional notation) what it means for set A to be 1) a subset of B, 2) equal to B, or 3) a proper subset of B.
- Memorize De Morgan's laws for sets. Compare with the De Morgan's law from logic. 1) Does it make sense that they share the same name? 2) Write the complements of
$$\bigcap_{i=1}^{100} A_i, \qquad \bigcup_{i=1}^{100} A_i.$$
- Write precisely what it means for x to be in the range of a function $f : A \to B$. Write precisely what it means for f to be 1) injective and 2) surjective.

1. (Ribet Spr13) If r, s, and t are real numbers, prove that the products rs, rt, and st are not all negative.

2. (Ribet Spr13) Consider the set of all sequences $\{a_n\}$ whose terms a_n are binary digits. (In other words, each a_n is 0 or 1.) Show that this set is uncountable.

3. (Ribet Spr15) Suppose that $f : A \to P(A)$ is a function from a set to its power set. Let
$$B = \{b \in A \mid b \notin f(b)\},$$
and let c be an element of A. Show that $f(c) \neq B$ by deriving a contradiction from the assumption $f(c) = B$.

4. (Sturmfels Spr09) Prove that 5 divides $n^5 - n$ whenever n is a positive integer.

5. (Sturmfels Spr09) The symmetric difference $A \oplus B$ of two sets A and B is the set containing those elements in either A or B but not in both A and B. Determine whether this operation is associative; that is, if A, B, and C are sets, does it follow that $A \oplus (B \oplus C) = (A \oplus B) \oplus C$?