Math 55: Discrete Mathematics
Williams, Spring 2018
GSI: Ai

WEEK 11: RELATIONS

1. (Ribet Sp13) Let A be the set of bit strings $a = a_1a_2...a_9$ of length 9. Let $R \subseteq A \times A$ be the set of pairs (a, b) such that $a_1 = b_1$ or $a_2 = b_2$. Decide whether or not the relation R is i) reflexive, ii) transitive, iii) symmetrix, iv) antisymmetric, v) an equivalence relation.

2. (Sturmfels Sp12) Give an example of a relation R such that its transitive closure R^* satisfies $R^* = R \cup R^2 \cup R^3$, but $R^* \neq R \cup R^2$.

3. (Sturmfels Sp09) How many relations are there on the set $A = \{1, 2, 3\}$ that are a) symmetric? b) reflexive and symmetric? c) neither reflexive nor irreflexive?

4. (Ribet F97) Suppose A is a finite set with at least two elements and that R is an equivalence relation on A. Show that there are distinct elements $a, a' \in A$ whose equivalence classes $[a]_R$ and $[a']_R$ have the same number of elements.