Math 55: Discrete Mathematics
Williams, Spring 2018
GSI: Ai

Week 10: Midterm Prep: Graded Homework Problems

- Step 0: (recommended) In each practice problem, think about what components would likely be worth points in a rubric, and know how to earn those points (if you erased everything but line x of your answer, would you get any points at all?)
- Example 1: In the chocolate bar induction proof, if you write "$P(n)$ = it takes $n-1$ breaks to break an n-sized chocolate bar," and nothing else, you might get 1 or 2 points. If you wrote "In induction, there is a base case and an inductive step," you would likely get 0 points.
- Example 2: If you wrote only "Base case: 0 breaks to break a 1-piece bar," likely 1-2 points. If you wrote only "4-piece bar takes 3 breaks, by checking two cases: in a row or in a square," many GSIs would give you 0 points.
- Step 1: Do the practice midterm as Prof. Williams recommended: 80 minutes.
- Step 2: Write perfect (think of a rubric) solutions to the practice midterm.
- Step 3: Look at the homework problems that Prof. Williams wanted graded (collected below, lightly modified). Know how to write perfect solutions.

1. 4.4-8. Show that an inverse of a modulo m, where a is an integer and $m > 2$ is a positive integer, does not exist if $gcd(a, m) > 1$.

2. 5.2-10. Determine how many breaks you make to separate an n-piece chocolate bar, and prove it.

3. 6.4-34. Use a path counting argument to show that n-choose-k = n-choose-$(n-k)$. (Probably too vague for midterm, but worth thinking about: If it were, where are the rubric points earned? I talked about this in class.)

4. 7.1-40. Suppose there are 5 doors in the Monty Hall problem: You choose 1 door, the host opens a losing door that you didn't choose. Probably of winning if you keep your door? Probability of winning if you switch to one of the 3 remaining doors randomly?

5. 7.3-10. Suppose 4% of patients have the flu. A test for the flu is accurate to the following extent: 95% of flu-patients test positive and 2% of healthy patients test positive. What is the probability that a patient testing positive for the flu has the flu? What is the probability that a patient testing negative for the flu has the flu?

6. 8.4-34. Use generating functions to solve the recurrence relation
$$a_k = 3a_{k-1} + 4^{k-1}$$
with initial condition $a_0 = 1$.