Math 55: Discrete Mathematics
Williams, Spring 2018
GSI: Ai

Week 1: Propositional Logic

Warm up questions:

- What are the relationships between a truth, a tautology, and a logical equivalence?
- Similarly, clarify the relation between an argument, a proof, and a rule of inference.
- Memorize De Morgan's laws. These are examples of what?

1. (Ribet Spr13) For each of these sets of premises, what relevant conclusions (if any) can be drawn?
   (a) All insects have six legs. Dragonflies are insects. Spiders eat dragonflies.
   (b) I am either dreaming or hallucinating. I am not dreaming. If I am hallucinating, I see elephants running down the road.

2. (Ribet Spr15) Use existential and universal quantifiers to express the statement "Everyone has exactly two biologial parents" using the propositional function $P(x, y)$, which represents "x is the biologial parents of y."

3. (Ribet Spr15) Decide whether or not the compound proposition
$$(\neg q \vee (p \to q)) \to \neg p$$
is a tautology.

4. (Ribet Spr15) Suppose that $f : A \to P(A)$ is a functio nfrom a set to its power set. Let
$$B = \{b \in A \mid b \notin f(b)\},$$
and let c be an element of A. Show that $f(c) \neq B$ by deriving a contradiction from the assumption $f(c) = B$.

5. (Sturmfels Spr09) Determine the truth value of each of these statements if the domain of each variable is the set of nonnegative integers:
   (a) $\exists\, x\ ((x^2 < 10) \wedge (|3 - x| > 2))$
   (b) $\forall\, x\ ((x \neq 4) \to (x - 5 > 1))$
   (c) $\forall\, x\ \exists\, y\ (x + y = 0)$
   (d) $\exists\, x\ \forall\, y\ (xy = 0)$.

6. (Sturmfels Spr12) Express the negations of each of these statements so that all negation symbols immediately precede predicates:
   (a) $\forall\, x\ \exists\, y\ \forall\, z\ T(x, y, z)$.
   (b) $\forall\, x\ \exists\, y\ P(x, y)\ \vee\ \forall\, x\ \exists\, y\ Q(x, y)$
   (c) $\forall\, x\ \exists\, y\ (P(x, y)\ \wedge\ \exists\, z\ R(x, y, z))$
   (d) $\forall\, x\ \exists\, y\ (P(x, y) \to Q(x, y))$.