Prelim Workshop
Summer 2018

Analysis Worksheet 6: Complex Analysis II

1. True or false: A function $f(z)$ analytic on $|z-a|<r$ and continuous on $|z-a|\le r$ extends, for some $\delta>0$, to a function analytic on $|z-a|<r+\delta$? Give a proof or a counterexample.

   SdS 5.6.2, Sp88. Types of singularities, branch points.

2. Let the function f be analytic in the region $|z|>1$ of the complex plane. Prove that if f is real valued on the interval $(1,\infty)$ of the real axis, then f is also real valued on the interval $(-\infty,-1)$.

   SdS 5.6.20, Fa92. Laurent expansion/Schwarz reflection.

3. Prove or disprove: If the function f is analytic in the entire complex plane, and if f maps every unbounded sequence to an unbounded sequence, then f is a polynomial.

   SdS 5.6.29, Sp87. Singularities at infinity.

4. For $0<a<b$, evaluate the integral
$$I=\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{|ae^{i\theta}-b|^4}\,d\theta.$$
   SdS 5.7.4, Fa99. Contour integrals.

5. Let f be an entire function such that
$$\int_0^{2\pi}|f(re^{i\theta})|^2\,d\theta\le Ar^{2k},\qquad(0<r<\infty),$$
   where k is a positive integer and A is a positive constant. Prove that f is a constant multiple of the function z^k.

   SdS 5.7.9, Sp01. Taylor expansion.

6. Let f be continuous on $\mathbb{C}$ and analytic on $\{z : Im(z)\neq 0\}$. Prove that f must be analytic on $\mathbb{C}$.

   SdS 5.7.6, Su77. Morera's Theorem.

7. Consider the polynomial
$$p(z)=z^5+z^3+5z^2+2.$$
   How many zeros (counting multiplicities) does p have in the annular region $1<|z|<2$?

   SdS 5.8.7, Fa83. Rouche's Theorem.