Math 55: Discrete Mathematics
Williams, Spring 2018
GSI: Ai

Week 1 Debrief: Propositional Logic

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Here are some thoughts, misconceptions, or errors I encountered a lot during my two sections:

1. Worksheet Problem 2 (the one with the parents):

   - **Some operations don't make sense with just any variable.** A lot of people tried to translate the "two parents" aspect of the problem into something about "$2x$." This expression doesn't have a natural interpretation because in this problem, x represents an object (a person), not a number. (For comparison, $x+y$ also doesn't make sense; what does it mean to add two people? In contrast, something like $x \neq y$ does make sense; it means that x is not the same person as y.)
   - **Different order of quantifiers mean different things.** Some people started their answer with
   $$\exists\ y\ \exists\ z\ \forall\ x\ ...$$
   This ends up saying that there exists two "supreme beings" y and z who are the parents of everyone. The correct answer would start
   $$\forall\ x\ \exists\ y\ \exists\ z\ ...$$
   which will end up saying that every person x has parents y and z, and so on, as desired.
   - **An answer which is just not right.** Here's a wrong answer:
   $$\forall\ x\ (\exists!\ y\ P(y,x)) \wedge (\exists!\ z\ P(z,x)).$$
   Here's one way to see that this is not right: Note that the expressions $\exists!\ y\ P(y,x)$ and $\exists!\ z\ P(z,x)$ are identical. (If you don't understand why, here's an analogy: Note that
   $$\sum_{i=1}^{2} i = \sum_{j=1}^{2} j = 3.$$
   The choice of i or j for my "dummy variable" doesn't matter. Similarly, the choice of y or z for the dummy variable above doesn't matter.) Since $blah \wedge blah$ is equivalent with just $blah$, the above answer is simply
   $$\forall\ x\ (\exists!\ y\ P(y,x))$$
   which is the sentence "Everyone has exactly one parent." From the English side, this is clearly not the same as the given problem.

2. Worksheet Problem 3 (the one with the truth table):

- **Do as little work as logically possible.** This problem was asking whether something is a tautology. For something to be a tautology, the "last column" should all be T. So as soon as you see an F in the last column, you can stop working and conclude, "not a tautology." Saves you a few seconds.