Convex integration for the Monge-Ampere equation in two dimensions

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Abstract: We discuss the dichotomy of rigidity vs. flexibility for the $\mathcal{C}^{1,\alpha}$ solutions to the Monge-Ampere equation in two dimensions:

$$\mathcal{D}et\nabla^2 v := -\frac{1}{2}\text{curl curl } (\nabla v \otimes \nabla v) = f \qquad \text{in } \Omega \subset \mathbb{R}^2. \tag{0.1}$$

Firstly, we show that below the regularity threshold $\alpha < 1/7$, the very weak $\mathcal{C}^{1,\alpha}(\bar{\Omega})$ solutions to (MA ??), are dense in the set of all continuous functions. This flexibility statement is a consequence of the convex integration h-principle, whereas we directly adapt the iteration method of Nash and Kuiper in order to construct the oscillatory solutions. Secondly, we prove that the same class of very weak solutions fails the above flexibility in the regularity regime $\alpha > 2/3$. Our interest in the regularity of Sobolev solutions to the Monge-Ampere equation is motivated by the variational description of shape formation, which I will also explain in the talk.

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4 : 10pm-5pm, 740 Evans Hall