Math 110

PROFESSOR KENNETH A. RIBET

Last Midterm Examination

April 3, 2014

9:40–11:00 AM, 105 Stanley Hall

Your NAME:

Your GSI:

Please put away all books, calculators, cell phones and other devices. You may consult a single two-sided sheet of notes. Please write carefully and clearly in *complete sentences*. Your explanations are your only representative when your work is being graded.

Vector spaces are over $\mathbf{F}$, where $\mathbf{F} = \mathbf{R}$ or $\mathbf{F} = \mathbf{C}$. They may be infinite-dimensional if there is no indication to the contrary. Eigenvectors are non-zero!

At the conclusion of the exam, hand your paper in to your GSI.

Problem	Your score	Possible points
1		6 points
2		6 points
3		6 points
4		6 points
5		6 points
Total:		30 points

As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.

1. Label each of the following assertions as TRUE or FALSE. Along with your answer provide a correct justification or counterexample.

a. If S and T are operators on a finite-dimensional $\mathbf{F}$-vector space V such that $ST = 0$, then $TS = 0$.

b. If S and T are operators on a finite-dimensional $\mathbf{F}$-vector space V such that $ST = I$, then $TS = I$.

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2. Label each of the following assertions as TRUE or FALSE. Along with your answer provide a correct justification or counterexample.

a. If T is an operator on an $\mathbf{R}$-vector space V such that $f(T) = 0$ for some real polynomial $f(x)$ of odd degree, then T has at least one eigenvalue on V.

b. If v_1, v_2 and v_3 are eigenvectors of T such that $v_3 = v_1 + v_2$, then all three vectors have the same eigenvalue.

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3. Let U be a subspace of an $\mathbf{F}$-vector space V. Assume that $(v_1 + U, \ldots, v_t + U)$ is a basis of V/U and that $(u_1, \ldots, u_s)$ is a basis of U. Prove that $(u_1, \ldots, u_s; v_1, \ldots, v_t)$ is a basis of V.

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4. Let T be an operator on an $\mathbf{F}$-vector space V. Assume that $(T - aI)(T - bI) = 0$, where a and b are scalars in $\mathbf{F}$. If T is not a scalar multiple of the identity operator, prove that a and b are eigenvalues of T.

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5. Suppose that $V \subset \mathbf{F}^n$ is a proper subspace (i.e., a subspace not equal to all of $\mathbf{F}^n$). Show that there are scalars $c_1, \ldots, c_n$, not all of which are zero, so that $c_1 a_1 + c_2 a_2 + \cdots + c_n a_n = 0$ for all vectors $(a_1, \ldots, a_n)$ in V.

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