Powers of Linear Forms

What is the dimension of the space of polynomials of a certain degree that are annihilated by certain powers of fixed vector fields? We obtain explicit formulas by computing sagbi bases of Cox-Nagata rings. For del Pezzo surfaces, these rings are presented by quadratic polynomials, and, for the blow-up of projective $n$-space at $n+3$ points, we get a beautiful connection between the Verlinde formula and phylogenetic algebraic geometry.