Name: ________________________________

SID: ________________________________

Neighbors: Please write the names of the students next to you (or “None”):

Left: __________________________________

Right: __________________________________

Section: Circle your discussion section below:

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<th>Room</th>
<th>GSI</th>
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<tr>
<td>MWF 8-9am</td>
<td>102</td>
<td>4 Evans</td>
<td>Jason Ferguson</td>
</tr>
<tr>
<td>MWF 9-10am</td>
<td>103</td>
<td>41 Evans</td>
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<td>MWF 10-11am</td>
<td>104</td>
<td>39 Evans</td>
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<td>MWF 11-12pm</td>
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<td>MWF 12-1pm</td>
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<td>MWF 1-2pm</td>
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<td>MWF 2-3pm</td>
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<td>MWF 3-4pm</td>
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Other/none, explain: ________________________________

Grading

1 / 4

2 / 4

3 / 6

4 / 6

5 / 6

6 / 6

7 / 6

8 / 6

/44

Instructions:

• Closed book: No notes, no books, no calculators.

• Exam time 80 minutes, do all of the problems.

• You must justify your answers for full credit.

• Write your answers in the space below each problem.

• If you need more space, use reverse side or scratch pages. Indicate clearly where to find your answers.

• You do not have to simplify your answers for any question. More specifically, we will accept answers in terms of any combination of [finite] sums, differences, products, quotients, polynomials, exponents, logs, absolute values, trig functions, inverse trig functions, factorials, $P(n, k)$, $C(n, k)$, $S(n, k)$, and $p_k(n)$. 
1. (4 points) You roll a die (possibly biased) 100 times, and 20 of the rolls land on 6. Using this, construct a 95% confidence interval for the probability $p$ for rolling a 6.

2. (4 points) $X$ is a random variable that gives $-4$, $0$, and $4$ as outputs with probabilities $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{4}$, respectively, and can give no other outputs. Find $E[X]$ and $\text{Var}[X]$. 
3. You independently toss three coins and count the number of heads, and repeat this experiment for a total of 120 times. You get 0 heads 12 times, 1 head 38 times, 2 heads 50 times, and 3 heads 20 times. Suppose you want to test the null hypothesis $H_0$ that all three coins are fair.

a) (3 points) Construct a table showing the observed frequencies and the expected frequencies under the null hypothesis $H_0$.

b) (3 points) What is the $\chi^2$-statistic for this data? Describe how to use this to determine if the null hypothesis can be rejected.
4. (6 points) Solve the recursion equation

\[ a_0 = 3, \quad a_1 = 0, \]

\[ a_n = p a_{n-1} + 2p^2 a_{n-2}, \quad n = 2, 3, 4, \ldots, \]

where \( p \) is a nonzero constant. Leave your answer in terms of \( p \).
5. (6 points) Evaluate the integral \( \int \frac{x^2 + 2x - 2}{x^3 + x^2} \, dx \).
6. (6 points) Solve the initial value problem \( ty'' + 2y' = 12t^2 \), \( y(1) = 1 \), \( y'(1) = 1 \).

*Hint:* Let \( z = y' \). Solve for \( z \), and then use \( z \) to solve for \( y \).
7. (6 points) Find all solutions, if any, to the differential equation $e^{-t}y' = (y + 1)^2$. 
8. **a)** (3 points) Show that \( y_1(t) = t \) and \( y_2(t) = e^{2t} \) are both solutions to the differential equation \((2t - 1)y'' - 4ty' + 4y = 0\).

**b)** (3 points) Solve the initial value problem

\[(2t - 1)y'' - 4ty' + 4y = 0, \quad y(0) = 1 \text{ and } y'(0) = -3.\]