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For each subset of the vector space of $2 \times 2$ matrices, determine whether or not it is a vector subspace & justify your answer.

a) $\{ (a, b, c, d) \mid a + b + c + d = 0 \}$

b) $\{ (a, b, c, d) \mid abc \cdot cd = 0 \}$
Let \( A = \begin{pmatrix} 2 & 2 \\ 1 & -2 \end{pmatrix} \). For each \( \vec{b} \), solve \( A\vec{x} = \vec{b} \) if possible; if it is not possible, find the least-squares solution to \( A\vec{x} = \vec{b} \) (i.e. find \( \vec{x} \) s.t. \( \|A\vec{x} - \vec{b}\| \) is as small as possible.)

(a) \( \vec{b} = \begin{pmatrix} 12 \\ 9 \\ 3 \end{pmatrix} \)

(b) \( \vec{b} = \begin{pmatrix} 12 \\ 9 \\ 5 \end{pmatrix} \)
Find an orthogonal matrix $P$ and a diagonal matrix $D$ such that

$\begin{pmatrix} 1 & 3 \\ 6 & 8 \end{pmatrix} = P^T D P$.

(Reminder: $A$ is orthogonal means $A^T A = I$.)
Recall from the homework that \( H_0(x)=1 \), \( H_1(x)=2x \), and \( H_2(x)=4x^2-2 \) are orthogonal with respect to the inner product \( \langle f, g \rangle = \int_{-\infty}^{\infty} e^{-x^2} f(x) g(x) \, dx \).

Compute \[
\frac{\int_{-\infty}^{\infty} e^{-x^2} x^2 (4x^2-2) \, dx}{\int_{-\infty}^{\infty} e^{-x^2} (4x^2-2)^2 \, dx}
\]
Find the general solution to the differential equation:

\[ y''(x) - y'(x) - 2y(x) = \cos x - \sin 2x \]
Find the Fourier series of
\[ f(x) = \begin{cases} 
2, & \pi > x \geq 0 \\
-2, & 0 > x > -\pi 
\end{cases} \]

What value does the Fourier series converge to at \( x = 0 \)?
Find a formal solution to the wave equation with the given initial-boundary conditions:

\[ \frac{\partial^2 u}{\partial x^2} = 64 \frac{\partial^2 u}{\partial x^2} \quad 0 < x < \pi, \quad t > 0 \]

\[ u(0,t) = u(\pi,t) = 0, \quad t > 0 \]

\[ u(x,0) = 2, \quad 0 < x < \pi \]

\[ \frac{\partial u}{\partial t}(t,0) = \sin 7x - \frac{1}{2} \sin 19x, \quad 0 < x < \pi \]