Math 1A (Summer 2009)
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Final Exam
14 August 2009

Name: ______________________________

Problem	Points
1	**/15**
2	**/12**
3	**/8**
4	**/15**
5	**/12**
6	**/10**
7	**/8**
Total	**/80**

Instruction: Welcome to your Final! Read **ALL** the questions and do easier problems first. There is no need to simplify your answers but you have to **show and explain your work**. Correct answer without any work/ explanation will not get you any points.

1. **(15 points)** A novice calculus student came up to you and said the following statements. You having gone through Math 1A is now an expert and realized that **ALL** his statements were **FALSE**. Be kind to this young chap and explain why each statement is false.

a) Since I know that $\lim_{x \to \pi} \frac{|\sin x|}{x - \pi}$ does not exist, then $f(x) = |\sin x|$ is not continuous at $x = \pi$.

b) I cannot calculate exactly the integral $\int_{-1}^{1} \frac{\sin x}{1 + x^2}\, dx$.

c) The function $f(x) = \int_0^x t^2 - 1\, dt$ is maximized at $x = 1$.

d) I can use l'Hospital rule to compute $\lim_{x \to \frac{\pi}{2}^+} (\tan(x))^{\cos x}$.

e) I know this comparison property of the integral:
Suppose f is continuous on $[a, b]$ where $a \leq b$. If $m \leq f(x) \leq M$ for $a \leq x \leq b$, then

$$m(b - a) \leq \int_a^b f(x)\, dx \leq M(b - a)$$

I know this property is useful to bound definite integrals, but I cannot guarantee that these numbers m and M would always exist.

2. (**12 points**) **INSTRUCTION:** Compute **exactly** the following three integrals. You have to use **each** of the following methods **at least once** in taking the integral: the definition of definite integral, substitution, the Fundamental Theorem of Calculus and interpreting the integral as area under the curve. *You may use more than one method in one problem*. Choose wisely!

These formulas may help:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

(a) $\int \frac{1-x}{\sqrt{1-x^2}}\,dx$

(b) $\int_0^2 (2-x^2)\,dx$

(c) Let $f(x) = \begin{cases} x+2 \text{ when } -2 \le x \le 0 \\ \sqrt{4-x^2} \text{ when } 0 < x \le 2 \end{cases}$

Find $\int_{-2}^{2} f(x)\,dx$.

3. (a) (**2 points) <u>Very carefully</u>** compute the following limits.

(i) $\lim_{x\to 2} x^2 - 4x + 7$

(ii) $\lim_{x\to 2} x^2$

(b) (**6 points**) Choose **<u>ONE</u>** of the limits above and prove using epsilon-delta definition of limit.

4. (**15 points**) Compute the following limits!

(a) $\lim_{x \to 0} \frac{\tan^2(x)}{x^2}$

(b) $\lim_{x \to \infty} \frac{x}{\sqrt{x^2+1}}$

(c) $\lim_{h \to 0} \frac{\int_{(x+h)}^{3(x+h)} t^2 \sin t \, dt - \int_{x}^{3x} t^2 \sin t \, dt}{h}$

5. (**12 points**) Sketch the function $f(x) = e^{1/x}$.

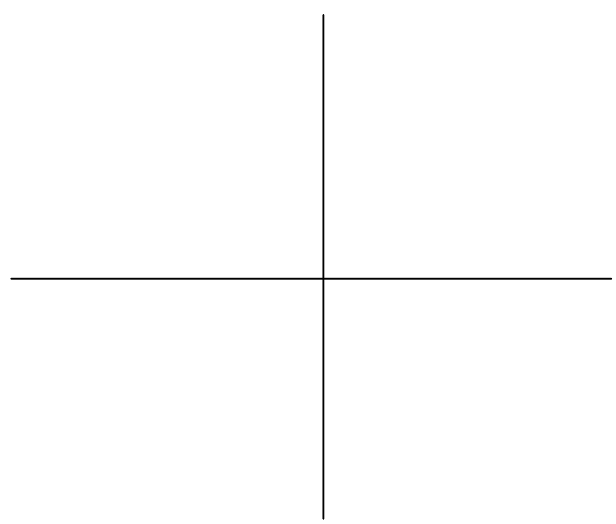
(a) Domain:

(b) Asymptotes:

(c) Critical points, intervals of increasing/ decreasing:

(d) Local extrema:

(e) Intervals of concavity:



(f) Inflection points:

(g) Graph:

(h) Range:

6. (a) **(5 points)** Find the point(s) on the parabola $y = x^2 - 4$ that is (are) closest to the point $(0, \frac{1}{2})$.

(b) **(5 points)** Draw a line connecting the point $(0, \frac{1}{2})$ and the point(s) you found in part (a). Find the area enclosed by your line(s) and the parabola.

7. (**8 points**) Omar didn't get *the girl*. Sad and frustrated he decided to run around the track at the gym. When he got to the track, one girl who was about to start running captured his attention. Omar hurried so they could start running together. They started at the same time, but then she started running faster. Omar sped up, but then she passed him again. Next thing he knew, the two of them were racing. At some point she noticed the finish line, and yelled out, "I don't go for losers!" Omar ran as fast as he could and the race ended in a tie. After the race she confidently came up to him and said, "I don't *mean* to be rude, but I only *value* smart guys. If you can prove the next thing I say, I'll go on a date with you." She then said, "We started the race at the same time, and the race ended in a tie, I claim that at some point during the race we were running at the same speed." She smiled at Omar and said, "Are you smart enough?" Use Calculus to help Omar! [*Hint*: Consider $f(t)$ and $g(t)$, the position functions of the two runners].