MESSAGE FROM THE CHAIR

By Jack Wagoner

Congratulations to all our 1994 Graduates! I hope you’ve enjoyed your experience here and I wish you well in achieving your new goals.

We continue to be delighted with the positive comments we receive following each issue of this newsletter. I especially want to thank all our former graduates for your warm responses and to invite you to keep this two-way communication flowing. We also welcome you to participate in any of the events occurring in the Department or Center which may be of interest.

It is gratifying to be able to report good news on several fronts:

Berkeley Mathematics had a strong showing in recent NSF fellowship awards. Graduating seniors awarded National Science Foundation Fellowships for graduate study in mathematics are Daniel Charles Isaksen and Catherine Helen O'Neil. Continuing graduate students here at Berkeley receiving NSF fellowships are Dimitri Y. Shlyaktenko and Monica Joy Vazirani. And former undergraduates now doing graduate work at other universities who won NSF fellowships are Stephen David Miller and Mihaela Vanderven.

Our faculty is truly outstanding and continues to win awards as in previous years. Assistant Professors Fraydoun Rezaakhaniou and Vera Serganova received Sloan Foundation Research fellowships for young faculty. Professor Paul Chernoff was inducted into the Golden Key National Honor Society. Professor Curtis T. McMullen received a Miller Research Fellowship, and Professor Marina Ratner won the John J. Carty Award for Advancement of Science. Professor Robert Solovay has just been elected to the American Academy of Arts and Sciences.

I am very happy to tell you that for the second year in a row Manager Carolyn Katz has received the Excellence in Management Award given by the Berkeley Staff Assembly. Many thanks to her and to all the staff for their continued, dedicated service and contributions to this Department.

This year we are celebrating National Mathematics Awareness Week, April 25-29, and the theme is mathematics and medicine. Our intent is to provide events of interest to the Bay Area Mathematics community. See more details elsewhere in this newsletter.

As usual our faculty recruitment activities for the coming year have been busy. We received an overwhelming response (over 900 applicants) to see pg. 2
Congratulations to Our '93 - '94 Graduates

Chair, cont. from pg. 1

our advertisements for a handful of positions. We plan to fill two new Morrey positions for two-year appointments beginning in 1994-95, bringing our total to six. These prestigious visiting assistant professor positions for outstanding young mathematicians were established in honor of Charles Morrey, Jr. They are funded by the University. Our Morrey Program provides the essential core of the Department's visiting teacher program, and it is critical for us to maintain these positions to teach courses and provide stimulation and fresh energy to our academic program.

Unfortunately, we now move on to gloomier news concerning budget cuts:

During each of the past four years, our visiting faculty program has further eroded, to the point where we are uncertain whether we will receive funding for any new post-doctoral teaching positions other than those needed to replace our faculty who accept the third early retirement offering (VERIP III) this coming July. This has been devastating to our academic program, as we have found it necessary for budget reasons to cut back on the diversity and breadth of our courses. We expect this pattern to continue in the coming year. In addition, we have had to eliminate all of our readers for lower division courses. While we have received steady support and some cushioning of cuts from Dean Price and Provost Christ, the College has experienced severe funding constraints and resources have been scarce. We, as well as many other campus departments and units, have been cut to the bone.

Our Department has done its best to protect our graduate students' financial support from extensive budget cuts. Again, out of necessity due to funding constraints, we are planning for significant decreases in our admittees for the graduate program. This year we will likely admit about 20 students (compared to our typical average of 60). This may be increased if we receive a positive response to our recent application for a Department of Education National Need Fellowship grant. Our primary concerns are to do our utmost to adequately support our students and to provide adequate coverage of our teaching needs. Each year we find these to be increasing challenges. Furthermore, we regret the adverse impact that the steadily increasing UC student fees have had on our students.

To end my message on a positive note:

I have been encouraged by the spirit of comraderie that has been with us throughout these years of financial cuts. Despite budget difficulties and the irreplaceable loss of many senior faculty to early retirement programs, we can look forward to rebuilding our department in the years to come. The campus administration is strongly committed to maintaining UC Berkeley as one of the best universities in the world. It is a time of great opportunity.
Congratulations to Assistant Professors Fraydoun Rezakhaniou and Vera Serganova on winning Alfred P. Sloan Research Fellowships in 1994.

Professor Rezakhaniou was born in Tehran, received his Ph.D. in Mathematics at New York University in 1989, and joined the UC Berkeley faculty in 1991. He studies probability theory and nonlinear partial differential equations. He is a particular expert on interacting stochastic particle systems and their macroscopic limiting behavior under scalings.

Professor Serganova was born in Moscow, received her Ph.D. in Mathematics at Moscow State University in 1983, and joined the UC Berkeley faculty in 1992. Her primarily studies representations of Lie superalgebras. She is continuing to investigate a general character formula for finite dimensional representations of simple Lie superalgebras.

Initiated in 1955, Sloan Research Fellowships aim to stimulate fundamental research by young scholars of outstanding promise at a time in their careers when their creative abilities are high and when federal and other support may be difficult to secure. These competitive grants are awarded to junior faculty members in chemistry, physics, mathematics, neurosciences, and economics.

In early February, 1994, our department received this announcement from the National Academy of Sciences:

“The John J. Carty Award for the Advancement of Science – a medal and a prize of $25,000 for noteworthy and distinguished accomplishments in any field of science within the charter of the Academy (the 1994 field is mathematics) – goes to Marina Ratner, Professor of Mathematics, Department of Mathematics, University of California, Berkeley, 'for her striking proof of the Raghunathan conjectures'. The award was established by the American Telephone & Telegraph Co. (AT&T).”

Marina was elected to the American Academy of Arts and Sciences in 1992 and to the National Academy of Science in 1993 and has been invited to deliver a plenary address at the 1994 International Congress of Mathematicians. For more information on her work, see the Berkeley Mathematics Newsletter of Fall, 1993.
DISTINGUISHED MILLER VISITOR, JEAN-PIERRE SERRE

By Kenneth Ribet

J.-P. Serre is one of the outstanding mathematicians of this century. He has made fundamental contributions in algebraic topology, commutative algebra, number theory, algebraic geometry, group theory, and the theory of modular forms. His many excellent books and fundamental articles have made him one of the most cited authors in all of mathematics. He was one of the first recipients of the Fields Medal (sometimes referred to as the equivalent of a Nobel Prize). He is a member of the French Academy (l’Académie des Sciences), and a foreign member of the National Academy of the US and other national academies. He has received honorary doctoral degrees from universities all over the world.

Serre is world famous as a lecturer, with an unrivaled ability to explain cutting-edge research with perfect clarity. His courses at the Collège de France have attracted large enthusiastic audiences, including mathematicians from all of France and from any number of foreign cities.

Serre has visited Berkeley only on rare occasions, and he has never before made an official visit to the U.C. campus. His most recent stay in our town was a month-long visit to the MSRI during the 1986–87 special year on arithmetic algebraic geometry. We are delighted that Professor Serre is able to join us this semester as a Miller Visiting Professor. Serre will visit for three weeks beginning April 18; he will speak in the departmental Colloquium on April 28.

ROBERT SOLOVAY ELECTED TO THE AMERICAN ACADEMY OF ARTS AND SCIENCES

Professor Robert Solovay was one of eight Berkeley faculty members elected this year as Fellows of the American Academy of Arts and Sciences. The election of 184 new Fellows and 26 Foreign Honorary Members for their distinguished contributions to science, scholarship, public affairs, and the arts was announced by the Academy on March 11.

Professor Solovay is widely regarded today as one of the several greatest logicians in the world. He was a leader in developing the simple concept of ‘forcing’ into a powerful, sophisticated tool for obtaining difficult results in set theory. Later he was a pioneer in the dramatic development of the theory of large cardinal numbers and its profound interconnection with the determinateness of infinite games.

Distinguished by its breadth as well as its depth, Solovay’s research has included important contributions to such diverse fields as differential topology, functional analysis, primality testing, and theoretical computer science. Within logic, although best known for his work in set theory, he has also made significant discoveries in recursive function theory, proof theory, modal logic, Boolean algebra, and the theory of definability.

The Academy of Arts and Sciences was founded in 1780 by John Adams and other leaders of the young United States “to cultivate every art and science which may tend to advance the interest, honor, dignity, and happiness of a free, independent, and virtuous people.”
MATH VISITORS

Kenneth Dykema received his Ph.D. from UC Berkeley in 1993. He is a highly regarded young mathematician whose doctoral dissertation focused on the study of free products of von Neumann algebras. His work was so highly praised that he was awarded a prestigious NSF Postdoctoral Fellowship in September, 1993. He is currently one of our Adjunct Assistant Professors for the Spring '94 semester.

Elinor Velasquez was a UC President's Fellow here at Berkeley from 1991-93. This followed the receipt of her Ph.D. from UC San Diego in 1991. Her dissertation research concerned the study of Radon transform in finite groups. For this work she was awarded an NSF Postdoctoral Fellow in September, 1993. She is currently an Adjunct Assistant Professor for the Spring '94 semester.

Sergio Fenley joined our department in September, 1993 as an NSF Postdoctoral Fellow. He received his Ph.D. from Princeton University in 1989. His research interests include foliations, 3-manifolds and Kleinian groups and more specifically the interaction between the foliation and geometric structures in hyperbolic 3-manifolds. Prior to coming to Berkeley, he was an Assistant Professor at Washington University in St. Louis, Missouri.

PAUL CHERNOFF ELECTED INTO GOLDEN KEY NATIONAL HONOR SOCIETY

Professor Paul Chernoff, well known for his outstanding teaching and popularity with students, has been named an Honorary Member of the Golden Key National Honor Society. Each year this student-run organization offers membership to those students ranking academically in the top 15% of the junior and senior classes at participating universities. Professor Chernoff's election was announced at the local chapter's annual induction ceremony on March 15. He was one of only three Berkeley faculty members thus honored.

The Golden Key Society was founded in 1977 with the goal of recognizing and encouraging scholastic achievement. It currently has chapters at over 200 universities and colleges throughout the United States and has awarded more than a million dollars in scholarships.
FACULTY REPORT 1993–94

By Jack Wagoner, Chair

To help alleviate the recent budget crisis, the University offered two very attractive VERIP early retirement programs in July 1991 and January 1993. In 1990-91 we had 67 regular faculty members. A total of thirteen people took VERIP I and VERIP II, and there have also been two deaths and two separations since 1991. While there have been additions to our ranks to bring the faculty strength back up to 59, we now expect another massive retirement of roughly 7 to 10 faculty in the VERIP III program this July. This will bring the size of the regular faculty down to around 50 and the number of current emeriti up to over 35. We greatly appreciate the long years of service to the department by faculty who have retired earlier and by faculty who may retire soon.

As mentioned in my Chair’s Message, the impact of retirements on the department has indeed been great. While many of the emeriti are still around and contribute to the department in a number of ways, such as advising graduate students and participating in seminars, we do miss their presence as full-time faculty engaged in research, teaching, and service. Budget cuts have not allowed us to hire sufficiently many visiting faculty to replace the loss of regular teaching faculty, and consequently we have been forced to reduce the variety and number of courses we offer at both the undergraduate and graduate levels. We now begin a long period of rebuilding the faculty back up to a level of about 67 which we feel is necessary to maintain Berkeley as one of the world’s foremost centers for mathematical research while at the same time fulfilling our equally important commitment to teaching within the framework of a large state university.

Included in our present size of 59 are the three new faculty who joined our department this year. They are Full Professors Richard Borcherds, Jenny Harrison, and Maxim Kontsevich. Many thanks, respectively, to Professors A. Ogg, C. Pugh, and N. Reshetikhin for their help in preparing the following thumbnail sketches.

Richard Borcherds

Richard Borcherds has been an undergraduate, a graduate student (with John Conway), a research fellow, and a lecturer at Cambridge University, before joining our Department last fall.

He was a Morrey Assistant Professor here during the 1987–88 year. In 1992 he received the Junior Whitehead Prize of the London Mathematical Society, and he was awarded the Prize of the City of Paris at the first European Congress of Mathematicians.

His work on infinite-dimensional algebras has had important consequences for finite groups, physics, and number theory (modular functions). His accomplishments include:

1) giving the correct definition of hyperbolic Lie algebras, providing a vast extension of Kac–Moody theory, and many fascinating new theta-identities (à la Weyl–Macdonald–Kac);

2) defining the general concept of vertex algebra, important for physics and mathematics, and making possible the construction by Frenkel, Lepowsky, and Meurman of the Monster vertex algebra (whose automorphism group is the Monster of Fischer–Griess, the “last” group, of order about $10^{54}$);

3) proving the moonshine conjectures of Conway and Norton, to the effect that certain series derived from the Monster vertex algebra are certain modular functions (the proof uses a version of the no-ghost theorem of string theory).

All three are illustrated by the (elementary but new) identity

$$j(p) - j(q) = \left(1 - \frac{1}{q}\right) \prod_{m,n = 1}^{\infty} (1 - p^m q^n)^{c(mn)}$$

for the modular invariant $j(q) = \frac{1}{q} + 744 + 196884q + \cdots = \sum c(n) q^n$. This is the denominator formula for a generalized Kac–Moody algebra which is acted on by the Monster.

Faculty, cont.

see pg. 7
Jenny Harrison

Born in Atlanta, Georgia, in 1949, Jenny Harrison received her A.B. from the University of Alabama in 1971, and her Ph.D. from the University of Warwick in 1975. Before joining our faculty in July 1993, she held faculty positions at UC Berkeley, Oxford University, and Yale University. She works on smoothness questions in the fields of dynamics and geometric integration theory.

Harrison's best known work in dynamics is her $C^2$ counter-example to the Seifert conjecture. The Seifert conjecture stated that any flow on the three sphere $S^3$ has a fixed point or a periodic orbit. Harrison constructed a $C^{2+\alpha}$ flow on $S^3$ with neither. The construction involves fractals and number theory. In particular, Harrison produced a fractal Jordan curve $Q \subset \mathbb{R}^2$ with Hausdorff dimension $1 + \alpha$ which is invariant under a $C^{2+\alpha}$ diffeomorphism $f : \mathbb{R}^2 \to \mathbb{R}^2$ and $f|_Q$ is topologically conjugate to a Denjoy diffeomorphism of the circle. (A Denjoy diffeomorphism has an irrational rotation number and has an invariant Cantor set. Denjoy showed that such diffeomorphisms $S^1 \to S^1$ of differentiability class $C^{1+\alpha}$ exist but they cannot be $C^2$. Denjoy's result is analogous to the Sard–Whitney theorem that the set of critical values of a $C^r$ function $f : \mathbb{R}^m \to \mathbb{R}^n$ is a zero set in $\mathbb{R}^n$ when $r \geq \max\{1, m-n+1\}$, but that this fails to be true when $r = m-n \geq 1$.)

The techniques she used in constructing $Q$ led Harrison to her recent work in geometric integration theory. It starts out from Whitney's version of Stokes' theorem for limits of chains respecting his "flat norm" and "sharp norm", $\flat$ and $\sharp$, and leads to an integration theory for certain types of fractals, using a new norm called the "$\lambda$-natural norm", $\lambda$. The ideas are described more fully in Harrison's article, *Stokes' Theorem for Non-smooth Chains*, which appeared in the October 1993 issue of the Bulletin of the AMS. As a consequence of her integration theory, Harrison finds a fractal version of Denjoy's result: if the rotation number of a $C^{1+\alpha}$ Denjoy diffeomorphism of the circle is sufficiently irrational (poorly approximated by rationals), then the dimension of its invariant Cantor set is at least $\alpha$.

Outside math, Harrison is an enthusiastic cellist in the Kensington Symphony Orchestra and the mother of a seven-year old boy.

Maxim Kontsevich

Maxim Kontsevich did his undergraduate work at Moscow University. His graduate work was started at Moscow and completed at the University of Bonn, where he received his Ph.D. in 1992. Before joining our faculty in 1993, he held research positions at the Max–Plank Institute, Harvard University, and the Institute for Advanced Study at Princeton.

M. Kontsevich is a mathematician with a variety of interests from mathematical physics to algebraic geometry. He has made a fundamental contribution to the intersection theory of moduli spaces of curves by computing explicitly generating functions for intersection numbers in terms of the matrix Airy integral. This is also known as a proof of the Witten conjecture. Another important part of his work is related to applications of graph cohomology to invariants of manifolds and to the algebraic deformation theory in a broad sense. His current research interests lie in the area related to the famous mirror conjecture for manifolds.

In 1992, Kontsevich was awarded the Otto Hahn Medaille by the Max–Plank Society, and he also received the Prize of the City of Paris at the First European Congress of Mathematicians.
THE GEOMETRY OF MOTION CONTROL

By Jerrold E. Marsden ¹

Our understanding of biological and robotic locomotion, attitude control of spacecraft, and other systems is increasingly based on geometric concepts. When an animal or a robot moves its joints in a periodic fashion, it can rotate or move forward. This leads to the general idea that when one variable in a system moves periodically, motion of the whole object can result. This can be used for control purposes; the position and attitude of a satellite, for example, are often controlled by periodic motions of parts of the satellite, such as spinning rotors.

A tool that has been used to describe these effects is the theory of connections, a notion that is extensively used in geometry, general relativity and other parts of theoretical physics. This approach, part of the general subject of geometric mechanics, is also useful in the study of the stability or instability of a system and in its bifurcations, that is, abrupt qualitative changes in its phase portrait as a parameter changes.

There are many interesting applications of mechanical locomotion generation. For example, the ideas have been used in the design of robotic devices that, when controlled, are capable of generating motion in confined spaces. Medical instruments have been designed that can wholly or partially move inside the patient and can thereby be used for non invasive medical operations. Some engineering applications, such as micromotors—for example, a Panasonic camera lens—have been based on these principles.

Geometric ideas have been used in mechanics for many things including how to design stabilizing feedback control systems in attitude dynamics and to study symmetric systems with rolling constraints. We will describe a geometric framework that helps us better understand the ideas of locomotion generation and motion control and that has proven useful in the applications.

A falling cat is able to execute a 180° reorientation, all the while having zero angular momentum. It achieves this rotation generation by manipulating its joints. This creates shape changes that also change its moment of inertia. Astronauts who wish to reorient themselves in a free space environment can similarly do so by means of shape changes—holding one of their legs straight, they can swivel it at the hip, moving their foot in a circle. When they have achieved the desired orientation, they stop their leg movement. Similar movements for robots and spacecraft can be controlled automatically to achieve desired objectives. Recently, new insights have been discovered using geometric ideas and because of this, one refers to the extra motion that is achieved by the name geometric phase.

The history of this phenomenon is an interesting story—we shall only mention a few highlights. The shift in the plane of swing of the Foucault pendulum as the earth rotates once each 24 hours is one of the earliest examples. Rotations of the plane of polarized light in wound optical fibers and anomalous spectral shifts in rotating molecules are others. Michael Berry, in the early 1980’s, emphasized the usefulness of geometry in explaining the effects. It was quickly realized that these phase shifts occur in both classical and quantum mechanics and, through the work of Marsden, Ratiu and Montgomery, it was found that they can be directly linked with symmetry. Even for classical examples like the rigid body, the analytical calculation of the geometric phase is a fascinating topic.

The theory of geometric phases has an interesting link with noneuclidean geometry. Hold your hand at arms length, but allow rotation in your shoulder joint so your fist can move on a sphere. Move your fist along three great circles, forming a triangle on the sphere and during the motion, keep your thumb “parallel”; that is, forming a fixed angle with the direction of motion. After completing the circuit around the triangle, your thumb will return rotated through an angle relative to its starting position. This angle (in radians) is given by $\Theta = \Delta - \pi$ where $\Delta$ is the sum of the angles of

¹A somewhat more mathematical version of this article with references for further reading is available from the author.
which can be made stable through intervention. In fact, bicycle riders do this everyday. For another example, a supersonic jet can fly in an unstable mode (with the wings swept forward), but nonetheless is stabilized by feedback controls. Flying in this mode has the advantage that one can execute tight turns with rather little effort—just turn off the controls! One of the interesting things is that the subjects that have come before, namely the use of connections in stability theory can be turned around to be used to find useful stabilizing controls, for example, how to control the onboard gyroscopes in a spacecraft to stabilize the otherwise unstable motion about the middle axis of a rigid body.

Another issue of importance in control theory is that of optimal control. Here one has a cost function (think of how much you have to pay to have a motion occur in a certain way). The question is not just if one can achieve a given motion but how to achieve it with the least cost. There are many well developed tools to attack this question, the best known of these being what is called the Pontryagin Maximum Principle that relates optimal trajectories to certain problems in Hamiltonian dynamics. In the context of problems like the falling cat, there is a remarkable consequence of Pontryagin’s Principle due to Richard Montgomery, namely that (relative to an appropriate cost function) the optimal trajectory in the base space is a trajectory of a Yang-Mills particle. The equations for a Yang-Mills particle are a generalization of the Lorentz equations for a particle with charge $e$ moving in a magnetic field $B$:

$$\frac{dv}{dt} = -\frac{e}{c} v \times B,$$

where $v$ is the velocity of the particle and where $c$ is the velocity of light. Connections again enter because the field $B$ is the curvature of a connection (the “mechanical connection”).

One of the interesting recent developments is that one can also use these geometric ideas for systems with rolling constraints. One of the differences with the examples presented so far is that in many examples with symmetry one has a conservation law, such as conservation of angular momentum. With rolling constraints, even in the presence of symmetry, such laws require serious reconsideration. Nevertheless, despite this basic difference, there has been the beginnings of a deeper understanding of how to transfer our knowledge from mechanical systems with angular momentum type constraints to those with rolling constraints. The use of connections has been one of the valuable tools in this endeavor. A recent paper of Bloch, Krishnaprasad, Marsden and Murray, as well as one of Lewis, Murray, Ostrowski and Burdick (a group at Caltech), has been developing this point of view. One of the systems they consider is the skateboard, which we alluded to earlier. This is a rich example that illustrates several of the ideas we have been discussing. Recall that with the skateboard, one can, by turning one’s feet, independently steer the front and back wheels—in the standard skateboard, these wheels are of course fixed to the frame. In addition, one can manipulate one’s own body and this motion is coupled to the motion of the skateboard itself.

One of the fascinating things about the skateboard is that one can generate locomotion without any foot pedaling. When the user’s feet and body are moved in a proper synchronous way, rotational and translational motion of the device is generated. The skateboard is simple enough so that one can explicitly study many aspects of it.
the triangle. The fact that \( \Theta \neq 0 \) is a basic fact of non-Euclidean geometry—in curved spaces, the sum of the angles of a triangle need not be \( \pi \) (i.e., 180°). This angle is related to the area \( A \) enclosed by the triangle by \( \Theta = A/r^2 \), where \( r \) is the radius of the sphere.

While explicit models of the falling cat have been studied both numerically and analytically, for example, by Kane, Shur, and Montgomery, some of the key features can be seen in simpler models. One of them, called the planar skater,

"An idea that has been quite fruitful in the investigation of mechanical systems is that of a geometric connection."

consists of three planar rigid rods connected by two frictionless pin joints; the rods are free to rotate as a whole and relative to each other. Again, one can generate overall rotational motion by controlling the relative joint angles, and this is explicitly revealed by utilizing conservation of angular momentum.

The examples presented so far are different from what one finds in other mechanical systems in one crucial aspect—the absence of constraints of rolling, sliding or contact. For example, when parking a car, the steering mechanism is manipulated and movement into the parking spot is generated using the rolling of the wheels on the road. When a human or a robot manipulates an object (imagine twirling an egg in your fingers), it can reorient the object through the rolling of its fingers on the object. This can be shown in a demonstration of Roger Brockett: roll your fingers in a rotating motion on a ball resting on a table—you will find that the ball reorients itself under your finger—you have generated rotational motion! The amount of rotation is again related to the amount of area you capture in the rotating motion.

Notice that in all these cases, we again have the central idea that cyclic motion in one set of variables (often called the internal variables) produces motion in another set (often called the group variables).

So far we have discussed the generation of rotational motion. However, one can also generate translational motion. For example, microorganisms and snakes generate translations using cyclic manipulation of their internal variables (work of Shapere, Wilczek, Burdick, etc.). In a superficial sense the reason is quite simple: translations are available as group variables and the controls are such that these variables are activated. Often translational motion and rotational motion are coupled in interesting ways, as in the skateboard, a modification of the familiar skateboard that allows the rider to independently rotate the front and back wheels by rotating their feet. This, together with the rotary motion of the rider’s body, allows both translational and rotational motion to be generated. With appropriate controls, desired motions can be generated.

There are similar links between vibratory motion and translational and rotational motion, as in the developments of micromotors (work of Roger Brockett) and in motion generation in animals, as in the generation of waves from coupled oscillators, in the swimming of fish, in insect and animal locomotion.

A central question is how should one design the controls—for example, how should one control motions of the internal variables so that the desired group (translational and rotational) motions are produced? To make progress on this question, one needs to understand the underlying mathematical structure of the systems more deeply.

An idea that has been quite fruitful in the investigation of mechanical systems is that of a geometric connection. In the classical setting of Riemannian and Lorentzian geometry (as developed by Einstein), the connections are called the Christoffel symbols and are used to describe how curved a space is; the curvature tensor of the space is constructed out of them. In other settings (following work of Cartan), the connection is what is responsible for a corrected measure of acceleration; for

"Control theory is closely tied to dynamical systems"

example if one is on a rotating merry-go-round, one needs to correct one’s measurements of acceleration and connections can be used to do this.

Connections can be associated with bundle mappings that project larger spaces called the bundle onto smaller ones called the base, or shape space. Vectors tangent to the larger space that project to zero are called vertical directions. The general definition of a connection is a specification of a vector space at each point, called horizontal directions that complements the set of vertical directions. Connections in this general sense are ubiquitous in geometry and physics. For example,
they are one of the main ingredients in the modern theory of elementary particles, and are the primary fields in Yang-Mills theory, a generalization of Maxwell's electromagnetic theory. In fact, in electromagnetism, the equation \( B = \nabla \times A \) for the magnetic field may be thought of as an expression for the curvature of the connection (or magnetic potential) \( A \).

In the example of parallel transport of the thumb around the sphere, the larger space is the space of all tangent vectors (if you like, velocity vectors) to the sphere, and this space projects to the sphere itself simply by projecting a vector to its point of attachment to the sphere. The horizontal directions at each point is the collection of acceleration vectors (tangent vectors to the velocity vectors) that, within the intrinsic geometry of the sphere, have zero acceleration; that is, the directions determined by great circles.

In the thumb example, we saw that going once around the triangle produces a net change in the orientation of the thumb. The thumb is parallel transported, that is, it moves in horizontal directions with respect to the connection. In general, according to a law of motion that has the general form

\[
\frac{dz}{dt} = f(z, \mu),
\]

where \( \mu \) includes other parameters of the system (masses, lengths of pendula, etc.). The equations themselves include things like Newton's second law, the Hodgkin-Huxley equations for the propagation of nerve impulses, Maxwell's equations for electrodynamics, etc. Many valuable concepts have developed around this idea, such as stability, instability, chaotic solutions, etc.

Control theory adds to this situation the idea that in many instances, one can intervene in the dynamics rather than passively watching it. For example, while Newton's equations govern the dynamics of a satellite, we may want to intervene by controlling the onboard gyroscopes. One way to describe this mathematically is by modifying \( f \) so that it becomes dependent on additional control variables \( u \) that can be functions of \( t, z \) and \( \mu \). Then the dynamical equation reads

\[
\frac{dz}{dt} = f(z, \mu, u(t, z, \mu))
\]

and the objective, naively stated, is to choose the function \( u \) itself to achieve the desired goals. The dependence of \( \mu \) on \( z \) is called feedback.

Two examples of what is meant by control theoretic goals are steering and stabilizability. Steering tries to find a control \( u \) that produces a solution \( z \) joining two given points. One imagines manipulating the control, much the way one drives a car, so that the desired final state is achieved. This type of question has received much development; two of the main themes that have emerged are the Lie algebraic techniques based on Jacobi-Lie brackets of vector fields (in driving a car, you can repeatedly make two alternating steering motions to produce a motion in a third direction) and the second based on the application of differential forms (a subject invented by Elie Cartan in the mid 1920s, whose

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"Connections can be used to find stabilizing controls"

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if we have a horizontal motion in a bundle, and if the corresponding motion in the base is periodic, then the bundle motion will undergo a phase shift between the beginning and the end of its path. This shift, represented by a motion in the vertical direction is often given by an element of a group, such as a rotation or translation group, and as such is called the geometric phase. In many examples, the base space is the control space in the sense that the path in the base can be chosen by suitable controls. This setting provides a framework for understanding our opening phrase: when one variable in a system moves in a periodic fashion, motion of the whole object can result. Here, the "motion of the whole object" is represented by the geometric phase. Associated with this notion are plenty of lovely theorems and calculational tools; for example, one of these shows how to calculate the geometric phase in terms of the integral of the curvature over an area enclosed by the closed curve on the base, which is one reason that areas commonly appear in geometric phase formulas.

Control theory is closely tied to dynamical systems in the following way. Dynamical systems is concerned with the time evolution of systems; one considers a point \( z \) in a phase space \( P \) moving ac-

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"One of the fascinating things about the skateboard is that one can generate locomotion without any foot pedaling."

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power is only now being significantly tapped in control theory).

The problem of stabilizability has also received much attention. Here the goal is to take a dynamic motion that might be unstable if left to itself, but see pg. 12
NEWS FROM MSRI

By Nancy Shaw

New Chairman of the Board

Elwyn Berlekamp, UC Berkeley Department of Mathematics, has recently been elected the new Chairman of the MSRI Board of Trustees. He replaces John Morgan, Columbia University, who has served in that capacity since 1989. In addition to his role on the Board, Professor Berlekamp has been organizing a summer workshop on Combinatorial Games to be held at MSRI during July 11-22, 1994.

Coming Events

The year-long program on Differential Geometry will continue through the spring; it has been joined by a second program on Dynamical Systems and Probabilistic Methods in Partial Differential Equations (PDE). In addition to the ongoing seminars, there will be two workshops:


There are three further events scheduled for the summer:


August 1-12, Summer Graduate Student Program in Hyperbolic Geometry (for students from sponsoring institutions). Organizers: David Epstein, Jane Gilman and Bill Thurston.

During 1994-95, there will be a full-year program on Automorphic Forms and a half-year program on Complex Dynamics and Hyperbolic Geometry.

1995-96 will feature a full-year program on Several Complex Variables and two half-year programs: Holomorphic Spaces in the fall, and Convex Geometry in the spring of 1996.

If you wish to subscribe to MSRI's weekly electronic seminar announcements, send email to majordomo@msri.org with the message: subscribe announcements.

For more details on future programs, workshops, and other events at MSRI, you can obtain a full listing of options by sending email to http://info.msri.org or gopher info.msri.org or ftp info.msri.org

analytically and so that numerical simulations of its motion are fairly easy to implement. On the other hand, the skateboard still has many of the essential features that one would want for more complex systems, such as its ability to generate locomotion. From the mathematical and mechanical point of view, it has a rich geometry and symmetry structure, which makes it attractive.

An essential feature of the skateboard that sets it apart from examples like the planar skater and the falling cat, is that while it has the symmetry group of rotations and translations of the plane, the total linear and angular momenta are not conserved due to its interaction with the ground through the rolling constraint. No matter what motions the joints of the planar skater or falling cat make, the linear and angular momentum are conserved. Instead of abandoning the ideas of linear and angular momentum for the skateboard, this suggests using equations for the linear and angular momentum; these can be solved in terms of the internal motion and then from this solution, the locomotion can be computed. This structured approach with the symmetry built into the procedure is very fruitful. Not only is it analytically and geometrically attractive, but this method is more computationally efficient than a direct simulation of the whole system.

cont. from pg. 11
Finally, in the 80’s, Hirsch began to apply his expertise in topology and dynamics to differential equations, especially competitive, cooperative and monotone systems. He discovered a simple and unexpected result with many applications, namely, that forward orbits of most initial values approach the set of equilibria, if they are bounded. Coworkers in this part of Hirsch’s mathematics include Hal Smith, Mary Lou Zeeman, Mallet–Paret and Paul Waltman.

As an outgrowth of his work in differential equations, Hirsch has begun to investigate the mathematics of neural nets. These have been used as models for cognitive processes such as memory, attention, motor control, etc. They are being implemented in software and hardware as practical parallel computing devices. Hirsch participates in an active, high-level group of neural net researchers who include Walter Freeman, Leon Chua and Steve Grossberg.

MATHEMATICS AWARENESS WEEK

This spring we celebrated national Mathematics Awareness Week from April 25-29. Our goal was to provide events of interest to the Bay Area mathematics community. We invited Berkeley Math alumni; high school and college teachers; and the campus community to join us in seminars, panel discussions, Math Library and MSRI tours, and a teachers’ conversation. The national event’s theme, “Mathematics in Medicine”, was reflected in poster displays, the Math Library exhibit, and Professor Jerrold Marsden’s CPAM lecture on the topic, “Geometric Mechanics and locomotion Generation.”

Several panels, library tours, and the special MSRI tea and tour were developed especially for our students.

Many thanks to those who helped organize or who participated in Mathematics Awareness Week. Special thanks to Bisi Agboola, Arlene Baxter, Lenore Blum, Jo Butterworth, Ricardo Cortez, Lana Furukawa, Juliana Lopez, Jerrold Marsden, Rondi Phillips, Graham Randall, Dick Stanley, and Elinor Velasquez; and all the others who so willingly volunteered their time and energy.

These events were a collaborative effort between the Berkeley Mathematics Department, Center for Pure & Applied Mathematics, Mathematics Library, Mathematical Sciences Research Institute (MSRI), and the Professional Development Program (PDP).
ASTR/MATH/STAT LIBRARY UPDATE

By Jo Butterworth

Current Technological Advances in the Library

MS Windows has been installed on our MathSci CD-ROM terminal (MathSci is Mathematical Reviews 1940–1993 on CD-ROM). This should make searching, saving, downloading, and FTP'ing your searches to your personal account even easier! The General Library announced InfoLib recently. This is the library gopher. It is now one of the menu items listed on the libraryUs public library terminals. You can find out all about the libraries on campus via this gopher.

Future Projects

Professor Ray Larson from CALUs School of Library and Information Studies recently received a DOE grant in support of a next-generation online catalog system called CHESHIRE. It is still in development, but when the system is ready for implementation, it will premiere in our library.

New Staff Member

Ms. Belle Thornton (bthornto@library) has joined our library as Circulation Supervisor/Operations Manager. She has many years of CAL library experience, as well as a strong commitment to public service. Please stop in to introduce yourself to her.

UC Alumni Use of the Library

Did you know that when you join the California Alumni Association (membership (510) 642-3706) for $40 per year it includes a LIBRARY CARD? This is a great bargain, because for any one other than faculty, staff and students, Library Cards cost $100 year. Of course, anyone can come into the library and use the material there, but to check out items, you must have a current library card.

Gifts to the Library

One of the best parts of being a librarian is receiving gifts for the library. Not only is it supportive of our collection, but it makes us feel good to know that so many people care. So far, in 1993/94 we have had more than twenty individual donors contribute 500 books and journals, and $2000 in cash. We would like to extend a most sincere THANK YOU to all of you who are helping us maintain our excellence!

WHAT IS THE NOETHERIAN RING?

By Leanne Robertson

The Noetherian Ring is a group of women graduate students, post docs, and professors in mathematics. We meet once a week for refreshments and an introductory level math talk, usually given by one of our members. Topics this semester have included C*-algebras, the dynamics of Newton transformations, torus orbits in flag manifolds, Lie algebras, Ramsey theory, Poncelet's theorem and invariant measure, and optical imaging.

Last semester we started a program to bring prominent woman mathematicians to Berkeley to give the Department Colloquium. So far, Leila Schneps, Ruth Charney, and Lai-Sang Young have visited. This program has been funded by generous faculty donations from both grants and private funds.

TIPS FOR COMPUTER-FRIENDLY USERS: HELP FINDING SEMINARS

By Leo Tenenblat

It is now even easier to obtain the schedule of seminars to be held in the Mathematics Department. The existing methods have been improved and several other user-friendly means of access have been implemented, enabling network users all over the world to find out about the seminars. For those with accounts at math.berkeleys.edu, the command "help Seminars" will provide the schedule. For everyone else connected to the internet, this service is available through finger:

%finger seminars@math.berkeleys.edu | more

For those familiar with "news" (under the newsgroup ucb.math), "gopher", "wais", or "mosaic", it is quite simple to reach this list on the server math.berkeleys.edu.
MANAGER’S REPORT

By Carolyn Katz

It’s been wonderful to hear from our Math alumni — please keep those letters coming. Our newsletter and invitation to lectures, seminars, and other activities are our way of saying “we’re thinking of you.”

Staff News

Angela Guess-Westbrooks, Assistant to the Chair, will be leaving the Math Department to move to the state of Washington, where her husband’s job will be transferred. She had been with the Department for four years and during that time she expanded her responsibilities and organized the Chair’s office procedures and systems efficiently. Her Chairs were grateful for her cheerful demeanor and willing assistance. They and faculty relied on her assistance in all areas related to academic personnel. Best wishes to Angela and her family in their new location.

A special thanks goes to Bibi Basha, who has filled in for the past six months during Angela’s personal leave of absence. Bibi has provided needed continuity to that office.

After ten years with the Math Department, Doris Smith, Supervisor of the Student Services Unit, has resigned to move on to a new career direction in the area of computer software documentation and training. Doris brought to the Department a special knowledge of math gained while she was a student here; she received her A.B. degree in 1984. In addition to her expertise in mathematics and curriculum development, Doris was known for her fast pace, her ability to juggle several priorities, her phenomenal memory, sense of humor, and dedication to the faculty and students of the Math Department. A repository of historical information gleaned over the years, Doris brought unique and special qualities to her position. She capably served as Acting Manager during a turbulent period in 1991-92, at which time she provided needed stability. She will be missed.

Thanks for Service

Janet Ynon, Graduate Assistant, was recently honored at our monthly Staff Meeting for her 20 years of service to the University and 13 years in the Math Department. Congratulations and our sincere appreciation go to Janet.

Congratulations to Students

Congratulations to our 1993–94 graduates. I know you’ve faced many challenges. I share your pride in your achievements and wish you the best. We’d like to send you future issues of this newsletter, so please keep us informed of your future location.

Thanks for Volunteering

I continue to be grateful for the initiative displayed by our students, and for their creative ideas. I’d like to acknowledge the following students who have spent time and energy making things happen to improve life for students:

Daniel Bernstein for his help in improving our administrative computing by setting up a special program for a mass alias for grad students. This alias will be used by departmental administrators and maintained by MGSA.

Paul Brown for his service as President of MGSA. I appreciate his valuable suggestions for improving services to graduate students and for his time in developing the graduate student directory database.

Ricardo Cortez for help in developing an informational brochure about the graduate program and an informational piece about the MOC; and for helping to make the MOC reception a success.

Concetta Gomez and Julie Mitchell for their help in organizing the Physical Sciences Women’s Day event held on April 8 under the coordination of L & S Affirmative Action Officer Colette Patt.

Steven Hillion for his thoughtful ideas on ways to improve the Summer Sessions program and for volunteering to help plan an orientation session for new instructors this summer.

Graham Randall for his dedicated service as President of MUSA and his cooperation on many joint student/departmental ventures.

I am grateful always for student, faculty, and staff support and their ideas which help us all make progress together.

CAREER DAY

The Mathematics Undergraduate Student Association (MUSA) is celebrating Mathematics Awareness Week with Career Day on April 28. Career Day is an annual event in which MUSA invites several professionals with mathematics backgrounds to discuss what opportunities are available to Berkeley’s mathematics graduates. Invited professionals include representatives from actuarial science, computer science, operations research, finance, teaching, and scientific research. Career Day is an excellent opportunity for students to learn from graduates’ experiences and to understand the importance of a mathematics degree in the “real” world. If you would like to participate in a future Career Day, please contact:

Mathematics Undergraduate Student Association  
Department of Mathematics  
University of California  
Berkeley, CA 94720  
or  
e-mail: musa@math.berkeley.edu.
The Berkeley Mathematics Lecture Notes Series is published by the Center for Pure and Applied Mathematics, University of California, Berkeley, CA 94720.

This series provides an opportunity for the mathematics faculty at UCB to make informal, graduate-level class lecture notes available to the mathematics community. The aim is to provide an opportunity to keep graduate class notes updated as well as to try out preliminary versions of new texts.

Two Methods For Ordering: (1) A copy of the order form is available over Internet by anonymous ftp from "math.berkeley.edu" in the directory Lecture_Notes. To access this file, execute these commands:

```
ftp math.berkeley.edu
login: anonymous
password: (your email address)
cd pub/Lecture_Notes
get orderform (choice of .tex or .txt)
```

These files are also available via the math.berkeley.edu gopher server.

OR

(2) Please write the number of copies on the line in front of each volume listed below that you wish to order and note the price for each volume. If you are a California customer, please add 8.25% sales tax to your subtotal. Then add $2.00 per volume for shipping and handling within the U.S. and $3.50 per volume for shipping and handling outside the U.S. Return this article with your payment.

With either method, send order form/article along with payment by check payable to U.C. Regents to Gail Yoshimoto, Berkeley Mathematics Lecture Notes Series, Center for Pure and Applied Mathematics, University of California, Berkeley, CA 94720.

If you have any questions, please contact Gail (510) 642-6426, e-mail: gail@math.berkeley.edu.

Berkeley Mathematics Lecture Notes Currently Available

- Vol. 1: Numerical Linear Algebra, by James Demmel ($12.00 ea.)
- Vol. 2: Lectures on Turbulence Theory, by Alexandre Chorin ($11.00 ea.)
- Vol. 3A: Partial Differential Equations, by Lawrence C. Evans (Chapters 1–5) ($13.00 ea.)
- Vol. 3B: Partial Differential Equations, by Lawrence C. Evans (Chapters 6–11) ($14.00 ea.)
- Vol. 4: Lectures on Topology and Analysis, by Paul Chernoff, and Notes on Measure and Integration in Locally Compact Spaces, by William Arveson ($11.00 ea.)
- Vol. 5: Prelim Workshop Lecture Notes, by David Cruz-Uribe ($11.00 ea.)
- Vol. 6: Applications of Global Analysis in Mathematical Physics, by Jerrold Marsden ($13.00 ea.)

1This volume is a reprint of the 1977 Publish-or-Perish Press text.
2This volume is a reprint of the 1974 Publish-or-Perish Press text.

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Student (to professor): It's unfair I lost 10 points on this exam just for missing a minus sign.
Professor: OK, I'll change your score by 10 points. (Changes score from 69 to 59.)
Student: But Professor, shouldn't that be a 79?
Professor: Plus or minus, what's the difference?
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ALUMNI UPDATES

Many of our alumni have written to us and we would like to share some of their news with you...

Yigal Arens (Ph.D. 1986): Sr. Research Scientist and Project Leader for the Information Sciences Institute (a computer science research institute) at the University of Southern California in Los Angeles, CA.

Anthony Attard (AB, 1978): Actuary for Life of Virginia in Richmond, VA.

Jonathan Block (AB, 1982) obtained his Ph.D. at Harvard, received a Sloan Fellowship at MIT, was awarded an NSF Young Investigators Award, taught at the University of Chicago as a visiting professor and is now an assistant professor in the Department of Mathematics at the University of Pennsylvania in Philadelphia.


William G. Chinn (AB, 1941), though retired, is an active member of the Northern California Section of the Mathematical Association of America, serving on the TYC Committee and the COMUM Groups.

Margaret (Meg) M. Fink (AB, 1975): Assistant Director of the Systems Unit in the Financial Aids Office here at our very own U.C. Berkeley.

Frederick M. Goodman (Ph.D. 1979): Professor in the Department of Mathematics at the University of Iowa.

David Gomberg (AB, 1969) obtained a Ph.D. in Statistics in 1987 and now works at UCSF Computer Center (ITS). (David says, “Ask me about WestCoast LIVE!” So, David, I’m asking? ...Editor).

Bill Harrison (AB, 1986) is currently a doctoral student in Computer Science at the University of Illinois at Urbana–Champaign. His area of interest is automated theorem-proving.

Bruce R. Johnson (AB in Math & Statistics in 1971; MS in Range Science in 1979; Ph.D. in Ecology in 1986 at UC Davis) works as Sr. Environmental Research Scientist for the California EPA, Department of Pesticide Regulation in Sacramento. Bruce is particularly interested in the application towards understanding environmental fate processes and fractal geometry.

Maurice Kaasa (MA, 1975) is currently a Ph.D. student in the Department of Geology at Texas A & M.

Chai Kam Kwong (AB, 1972): Sr. Education Officer in the Education Department in Hong Kong.

Daniel Lieman (AB, 1987; MS, 1988 and Ph.D. 1992 at Brown University, RI) while at UC Berkeley received a Regents’/Chancellor’s Scholarship, the Edward Kraft Award for Academic Achievement, and won first place in the COMAP National Competition in Mathematical Modeling. At Brown University, Daniel was awarded the President’s Award for Excellence in Teaching, and a University Fellowship. He was also a Sloan Dissertation Fellowship Nominee, and an NSF Graduate Fellowship Honorable Mention. From 1992–93, Daniel received a Postdoctoral Fellowship and worked at MSRI in Berkeley. Daniel is currently a Ritt Assistant Professor at Columbia University, NY.

Walt Loew (AB, 1971): Software developer at ASK Group, Inc. in Alameda, CA.

Yong–Geun OH (Ph.D. 1988): Assistant Professor in the Department of Mathematics at the University of Wisconsin in Madison.

James R. Otto (MA, 1968): Currently a Ph.D. candidate and faculty lecturer in Mathematics at McGill University, Montreal, Quebec, Canada, expecting to graduate end of this summer. (Bon Courage, James! — Editor).

James T. Panttaja (AB, 1972 in both Math & Computer Science; MA, 1974) is Vice President of Panttaja Consulting Group, Inc., specializing in the development of relational database applications.


Jerry L. Yost (MA, 1973): Manager, International Distribution of First Pacific Networks in San Jose, CA.

Zachary M. Franco (Ph.D. 1990): Assistant Professor at the Department of Mathematics, Texas A & M University in Kingsville, TX.

I extend deepest sympathy to the DeLemus family for the loss of their son, Mark who died September 6, 1989 in Seattle while undergoing preparatory treatment for a bone marrow transplant for leukemia. Mark received his Masters Degree in Mathematics in 1982 from U.C. Berkeley. Like me, Mark was a “local”, having lived down the street from my own childhood home in Mountain View, California. His family now lives in Palo Alto, California.

— The Editor.
BABY NEWS

This semester we have “baby news”. We extend warm congratulations to two faculty and two graduate students of our mathematics community who are proud parents of “our future”!

Ph.D. student, Dong Yan and his wife, Meei-Chyi Guo had a baby girl on December 9, 1993. Her name is Christine T. Yan.

Vera Serganova, a new member of our faculty, welcomed in our last newsletter issue (Fall ’93), gave birth to a daughter on August 17, 1993. Marina Zakharevich was 9 lbs, 5 oz. and 21 and 3/4 inches at birth.

Nicolaï Reshetikhin, a member of the Mathematics faculty since July, 1991, is another proud father of a daughter, Anastasia Lily, born January 29, 1994. Anastasia weighed 8 lbs, 3 oz. and was 20 and 3/4 inches at birth.

The Department of Mathematics congratulates these new parents!

The Department of Mathematics congratulates these new parents!
ALUMNI AND FRIENDS NEWS AND UPDATE FORM

(Please type or print)

NAME:
Last ___________ First ___________ MI ___________

DEGREE & YEAR GRADUATED: ________________________

ADDRESS CORRECTION:

WORK - Position:
Institution or Company:
Location:

PERSONAL AND PROFESSIONAL NEWS: (Please type or print, using a separate sheet, if necessary.)

IDEAS FOR OUR NEWSLETTER:
What items in this issue were of particular interest to you?

What other types of articles or information would you like included in future issues?

OTHER COMMENTS:

Thank you for taking the time to help us plan for our next issue. Please return this form to Editor Rondi Phillips, Department of Mathematics, Rm 968 Evans Hall, University of California, Berkeley, CA 94720. (FAX number (510) 642-8204; TEL. (510) 642-4024; EMAIL: rondi@math.berkeley.edu)