When a Riemannian manifold $M$ has a smooth metric tensor, there is a successful theory of how the propagation of waves, solutions to $u_{tt} - \Delta u = 0$ on $\times M$, is well described in terms of the bicharacteristic flow. Natural classes of Riemannian manifolds arise for which even the nature of the bicharacteristic flow becomes rather subtle, but all the same one can treat it, and obtain results on the propagation of singularities for waves. The analysis involves a mixture of linear and nonlinear PDE.