Please check that you have all 5 pages of this exam booklet. Write your name on each page. As you turn through the pages, look for the easy questions—do them first. Remember that this exam is only 50 minutes long.

- You need not simplify your answers unless you are specifically asked to do so.
- It is essential to write legibly and show your work.
- If your work is absent or illegible, and your answer is not perfectly correct, then no partial credit can be awarded.
- Completely correct answers which are given without justification may receive little or no credit.
- Be especially sure to provide some explanation when you do the “true or false” problems.

During this exam, you are not allowed to use calculators or consult your notes or books.

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<th>Problem</th>
<th>Maximum</th>
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At the conclusion of the exam, hand in this exam paper to your TA.
1a (5 points). True or false: If a function $f$ maps a set to itself, then $f$ is one-to-one if and only if $f$ is onto.

1b (5 points). True or false: If $p$ and $q$ are propositions, then $\neg p \rightarrow q$ and $p \land q$ are equivalent.
2a (5 points). In the sleepy town of Cognac, 33 cars have license plates containing TI, while 109 have license plates containing the string NY. Suppose that 115 cars have license plates containing either TI or NY. How many cars have plates with TI or NY, but not both?

2b (8 points). Use the Euclidean algorithm to solve the congruence $5x \equiv 1 \pmod{13}$. 
3 (10 points). The celebrated Ribonacci numbers $R_n$ are defined as follows:

\[ R_0 = 0; \quad R_1 = 1; \quad R_n = 3R_{n-1} - 2R_{n-2} \text{ for } n \geq 2. \]

Prove that $R_n = 2^n - 1$ for all $n \geq 0$. 
4a (5 points). True or false: If $f(x) = \log_\pi x$ and $g(x) = \log_{10} x$, then $f(x) = O(g(x))$.

4b (7 points). Find a non-negative integer $x$ satisfying the two congruences

\[
\begin{align*}
    x &\equiv 15 \pmod{18} \\
    x &\equiv 47 \pmod{50}.
\end{align*}
\]

[Hint: first find a solution with $x$ negative.]