Math 32, Spring 2006
Instructor: Alex Diesl
May 12, 2006

Final Exam

Your Name: ____________________________
SID: ________________________________
Section: ______________________________

Directions:

This is a 170 minute exam. Please do not start until instructed to, and please stop working when time is up. Read directions, work carefully and check your answers. Make sure that your final answer is circled (or in some other way identified). If you are running out of room, use the back of the page you are working on and so indicate. If you need scratch paper or have a question, please raise your hand. Be advised, however, that we may not be able to answer your question. We will, however, give you as much scratch paper as you want.
1. (*5 points*) Pick any 5 angles $\theta$ (in radians) in the interval $[0, \frac{\pi}{2}]$. Fill in the table below with each angle, its cosine, and its sine.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\cos(\theta)$</th>
<th>$\sin(\theta)$</th>
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<tbody>
<tr>
<td>1.</td>
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<td>4.</td>
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<td>5.</td>
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</table>

2. (*5 points*) Sketch a graph of $y = \frac{1}{x-1} - 1$. 
3. (8 points) Solve the following system for $x$, $y$ and $z$:

\[
\begin{align*}
2x &- y + z = -3 \\
2y &- z = 11 \\
x &- 3y - 5z = 5
\end{align*}
\]
4. Let \( p(x) = x^4 - x^3 - 2x^2 + 6x - 4 \).

(a) (2 points) List all possible rational roots of \( p(x) \).

(b) (4 points) Find all (real and complex) solutions to the equation \( x^4 - x^3 - 2x^2 + 6x - 4 = 0 \).
(c) (4 points) Solve the inequality for \( x \): \( x^4 - x^3 - 2x^2 + 6x - 4 \leq 0 \). Write the answer in interval notation.
5. (a) (5 points) Find all pairs \((x, y)\) that solve the following system:

\[ y = x^3 + 2x^2 - 5x - 6 \]
\[ y = -x^2 + 8x + 9 \]
(b) (5 points) Graph \( y = x^3 + 2x^2 - 5x - 6 \) and \( y = -x^2 + 8x + 9 \) on the same set of axes. Label all intercepts and any points of intersection of the graphs.
6. (8 points) Find the domain of the given function. Write the answer in interval notation.

\[ F(x) = \frac{\log(1 - x^2)}{\sin^{-1}(x) - \frac{\pi}{6}} \]
7. (8 points) Let $f(x) = \cos(x)$ and $g(x) = 2x$. Graph $y = g(f(x))$ and $y = f(g(x))$ on the same set of axes. Label at least 5 points on each graph.
8. (5 points) Write down a quadratic polynomial that has $3 + i$ and its complex conjugate as its roots.

9. (5 points) Solve for $x$: $\log_3(x + 6) + \log_3(x) = 3$
10. (a) (3 points) The points $A$ and $B$ have polar coordinates $(2, \frac{-\pi}{3})$ and $(\sqrt{2}, \frac{3\pi}{4})$, respectively. Give the rectangular coordinates of $A$ and $B$.

(b) (3 points) The point $C$ has rectangular coordinates $(-1, -1)$. Write the polar coordinates for $C$ in four distinct ways.

(c) (2 points) Sketch the polar graph $r = 3$. 
11. Evaluate.

(a) (2 points) \( \sin(\frac{7\pi}{6}) = \) 

(b) (2 points) \( \cos^{-1}(-\frac{1}{2}) = \) 

(c) (2 points) \( \log_4(32) = \) 

(d) (2 points) \( \sin^{-1}(\sin(\frac{5\pi}{3})) = \)

Extra Credit (2 points) Tell me what you have learned about mathematical induction. You are encouraged to use quotes from lecture.