Math 121B midterm, 2004 April 1.  2nd midterm  R. Borcherds

Please make sure that your name is on everything you hand in.
You are allowed calculators and 1 page of notes.
All questions have about the same number of marks.

1. Solve the following differential equation by the method of Frobenius
(generalized power series):

\[ x^2 y'' - 6y = 0. \]

2. Express \( \frac{d}{dx} J_0(x) \) in terms of \( J_1(x) \), using the definition

\[ J_p(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x/2)^{2n+p}}{n!(n+p)!}. \]

3. Use the relation

\[ \exp(2xh - h^2) = \sum_{n=0}^{\infty} \frac{H_n(x)h^n}{n!} \]

to calculate the Hermite polynomials \( H_0, H_1, H_2, \) and \( H_3 \). What is the coefficient of \( x^n \) of \( H_n(x) \)?

4. The Laguerre differential equation is

\[ xy'' + (1-x)y' + py = 0. \]

Find the polynomial solution \( L_p(x) \) with constant term 1 for \( p = 3 \).

5. A bar of length \( \pi \) with insulated sides is initially at a temperature of
1. Starting at time \( t = 0 \), the ends are held at a temperature of 0.
Find the temperature distribution \( T(x, t) \) in the bar at time \( t \). The
temperature \( T \) satisfies the heat equation

\[ \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}. \]