April 14, 2004  Math 118  M. Rieffel
Second Midterm Exam

State your answers clearly and fully (with whole sentences, please). Include all your work.
(Total points = 40.)

1. For each integer \( n \) let \( V_n \) and \( W_n \) be the usual subspaces of \( L^2(\mathbb{R}) \) coming from the Haar scaling function and wavelet (for dilation by 2).

   a) Explain carefully how the \( V_n \)'s and \( W_n \)'s are obtained.

   b) Exhibit explicitly the Haar wavelet orthonormal basis for \( W_2 \).

   c) For each \( n \) let \( X_n \) be the subspace of \( W_n \) consisting of those functions which have value 0 outside the interval \([0, 5]\). Determine the dimensions of \( X_1 \) and \( X_{-1} \) by finding bases for them. Justify your answer.

2. Fix a positive integer \( N \), and let \( x \) and \( y \) be complex-valued functions on \( \{0, 1, ..., N-1\} \).

   a) Define what is meant by the (discrete) Fourier transform, \( \hat{x} \), of \( x \).

   b) Define the convolution \( x*y \) of \( x \) and \( y \).

   c) Show that \( (x*y)^\wedge = \hat{x} \hat{y} \), the pointwise product.