GEBA

MATH 110 - FINAL 5/21/2004

1. Let $A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix}$. Test A for diagonalizability and if it is diagonalizable, find an invertible matrix Q and a diagonal one D such that $Q^{-1}AQ = D$.

2. Solve the following system by Gauss elimination:

$$\begin{cases} x_1 + 2x_2 - x_3 + x_4 = 2 \\ 2x_1 + x_2 + x_3 - x_4 = 3 \\ x_1 + 2x_2 - 3x_3 + 2x_4 = 2 \end{cases}$$

3. Find the minimal solution for the system:

$$x + 2y - z = 12$$

4. Let $V = C([-1,1])$, the real inner product space of the continuous functions defined on $[-1,1]$, with the inner product:

$$< f, g >= \int_{-1}^{1} f(t)g(t)\,dt$$

Consider $W = P_{\leq 2}(\mathbb{R}) \subset V$, the subspace of polynomials of degree at most 2 with real coefficients.

i) Find an orthonormal basis for W.

ii) Let $h \in V$, $h(t) = e^t$, $\forall t \in [-1,1]$. Using i), find the best approximation of h in W.

5. Let V be an inner product space and $W \subset V$ a finite dimensional subspace of it. From theory we know that $V = W \oplus W^{\perp}$. We define $U : V \to V$ in the following way:

$$U(v) = v_1 - v_2$$

where $v = v_1 + v_2$ is the unique decomposition of v according to $V = W \oplus W^{\perp}$.

Prove that U is a self-adjoint, unitary linear operator.

6. Let $A \in M_{n\times n}(\mathbb{C})$ and consider $f(t) = det(A - tI_n)$ its characteristic polynomial. Prove that $f(A) = O_n$.

7. Prove that $det(A^2 + I_n) \geq 0, \forall A \in M_{n\times n}(\mathbb{R})$.