1. Let \( A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix} \). Test \( A \) for diagonalizability and if it is diagonalizable, find an invertible matrix \( Q \) and a diagonal one \( D \) such that \( Q^{-1}AQ = D \).

2. Solve the following system by Gauss elimination:
\[
\begin{align*}
x_1 + 2x_2 - x_3 + x_4 &= 2 \\
2x_1 + x_2 + x_3 - x_4 &= 3 \\
x_1 + 2x_2 - 3x_3 + 2x_4 &= 2
\end{align*}
\]

3. Find the minimal solution for the system:
\[
x + 2y - z = 12
\]

4. Let \( V = C([-1, 1]) \), the real inner product space of the continuous functions defined on \([-1, 1]\), with the inner product:
\[
<f, g> = \int_{-1}^{1} f(t)g(t) \, dt
\]
Consider \( W = P_{\leq 2}(\mathbb{R}) \subseteq V \), the subspace of polynomials of degree at most 2 with real coefficients.

i) Find an orthonormal basis for \( W \).

ii) Let \( h \in V \), \( h(t) = e^t \), \( \forall t \in [-1, 1] \). Using i), find the best approximation of \( h \) in \( W \).

5. Let \( V \) be an inner product space and \( W \subseteq V \) a finite dimensional subspace of it. From theory we know that \( V = W \oplus W^\perp \). We define \( U : V \to V \) in the following way:
\[
U(v) = v_1 - v_2
\]
where \( v = v_1 + v_2 \) is the unique decomposition of \( v \) according to \( V = W \oplus W^\perp \). Prove that \( U \) is a self-adjoint, unitary linear operator.

6. Let \( A \in M_{n \times n}(\mathbb{C}) \) and consider \( f(t) = \text{det}(A - tI_n) \) its characteristic polynomial. Prove that \( f(A) = O_n \).

7. Prove that \( \text{det}(A^2 + I_n) \geq 0 \), \( \forall A \in M_{n \times n}(\mathbb{R}) \).