J. Strain

Math 128A. Midterm #2. 4 April 2003.
Choose 3 of 4 problems:

1. Assume a numerical approximation $N$ to a quantity $M$ satisfies

$$N = M + 3h + 5h^2 + O(h^3) \text{ as } h \to 0,$$

and we know three specific values

$N(h) = N_1$, $N(h/2) = N_2$, $N(h/3) = N_3$.

Construct an approximation $N_{123}$ with

$N_{123}(h) = N + O(h^3)$.

2. The quadrature formula

$$\int_0^1 f(x) \, dx = c_1 f(-1) + c_0 f(0) + c_1 f(1)$$

has degree of precision (accuracy) 2.
Find $c_1$, $c_0$, and $c_1$. For what $p$ is the
error given by

$K \frac{f^{(p)}(1)}{3}$?
3. Suppose \( c_1, c_0, \) and \( c_1 \) are numbers such that the quadrature formula
\[
\int_a^b f(x)\,dx = c_1 f(a) + c_0 f(b) + c_1 f(1).
\]
has precision 2.

a) Find weights \( d_0, d_1, d_2 \) such that the quadrature formula
\[
\int_a^b f(x)\,dx = d_0 f(a) + d_1 f(b) + d_2 f(2)
\]
has precision 2.

b) Find a compound rule involving the \( c_i \)'s and \( d_i \)'s such that
\[
\int_a^b f(x)\,dx = \sum_{j=0}^{\infty} w_j f(jh) \quad (h = \frac{1}{n})
\]
\[
+ O(h^3) \quad \text{as } h \to 0.
\]
Express the weights \( w_j \) in terms of \( c_i \)'s and \( d_i \)'s.

4. a) Show that \( y' = y, \ y(0) = 1 \) has a unique solution \( y(t) \) on the interval \([0, 1]\).

b) Derive a Taylor method of order two for the initial value problem in (a).