1. Use a Laplace transform to solve
   \[ y' + y = \pi e^{-t}, \quad y(0) = 1. \]

2. Solve the heat eqn for a semi-infinite rod insulated at one end:
   \[ u_t = u_{xx}, \quad x > 0, \ t > 0 \]
   \[ u_x(0, t) = 0, \quad t > 0 \]
   \[ u(x, 0) = e^{-x}, \quad x > 0 \]
   leave your answer as an integral.

3. Evaluate the integral
   \[ \int_0^{2\pi} \frac{d\theta}{1 - r \sin \theta}, \quad r < 1 \]
   by contour integration.
4. Evaluate the integral
\[ \int_{-\infty}^{\infty} \frac{e^{ix}}{1 + x^4} \, dx \]
by contour integration. Assume \( x \in \mathbb{R} \).

5. Use Lagrange multipliers to find the closest points to the origin on the surface
\[ xy^2 = 8. \]

6. Suppose the functions
\[ xe^u \cos v + u e^v \sin v = 1 \]
\[ x^2 + y^2 + u^2 + v^2 = 2 \]
determine any two of \( x, y, u \) and \( v \) in terms of the other two. Evaluate
\( \left( \frac{\partial u}{\partial x} \right)_y \) and \( \left( \frac{\partial u}{\partial v} \right)_x \)
and use it for \( x, y, u \) and \( v \).