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# Math 54M Second Midterm

*80 minutes*

Spring 2001

1. (15 points) Let  $A = \begin{bmatrix} 3 & 18 & 10 & 2 & 7 \\ 1 & 6 & 2 & -3 & -4 \\ 2 & 12 & 8 & 5 & 11 \\ -1 & -6 & 0 & 10 & 7 \\ 6 & 36 & 12 & -21 & -11 \end{bmatrix}$ . Find a basis for:

(a).  $\text{RS}(A)$ (b).  $\text{CS}(A)$ 

2. (10 points) Find the coordinates of  $(1, 2, -8)$  relative to the basis

$$B = \{(-1, 3, 1), (2, -4, -3), (-3, 8, 2)\}.$$

3. (15 points) Find the equation of the line that best fits the points

$$(1, 2), (2, 3), (3, 5), (4, 8)$$

in the least-squares sense.

4. (15 points) Find an orthonormal basis for the subspace of  $\mathbb{R}^4$  spanned by the vectors

$$(1, 2, 3, 4), (0, 6, 4, 9), (-1, 10, 5, 14).$$

5. (15 points) Find the determinant of the matrix  $\begin{bmatrix} 1 & 2 & 7 & -5 \\ 2 & 6 & 11 & 1 \\ -1 & 0 & -6 & 13 \\ -2 & 0 & -15 & 33 \end{bmatrix}$ .

6. (15 points) Let  $A = \begin{bmatrix} 3 & 0 & 0 \\ -6 & 3 & -2 \\ 6 & 0 & 5 \end{bmatrix}$ .

(a). It has a double eigenvalue  $\lambda = 3$ . Find a basis for the corresponding eigenspace.(b). Find the third eigenvalue of  $A$ .

7. (15 points) Let  $A = \begin{bmatrix} 7 & -6 \\ 3 & -2 \end{bmatrix}$ . Find a  $2 \times 2$  matrix  $B$  such that  $B^2 = A$ .

**[Hint:** How would the eigenvalues and eigenvectors of  $A$  be related to those of  $B$ ?]