1. (54 points, 9 points apiece) Find the following. If an expression is undefined, say so.

(a) $\frac{dy}{dx}$, where $x = 2 \sin(e^t)$, $y = 5 \cos(e^t)$. Express your answer as a function of $t$.

(b) The length of the space curve given by the parametric equations $x = 2e^t$, $y = e^{2t}$, $z = t$ ($-1 \leq t \leq 1$).

(c) $\lim_{(x,y)\to(0,0)} (|x|+2)/(|y|+7)$.

(d) The equation of the plane tangent to the surface $z = (x^2 + y)^{1/2}$ at the point where $x = 3$, $y = 7$.

(e) $\frac{\partial^2}{\partial x \partial y} f(xy^2)$ where $f$ is a differentiable function. (Express your answer in terms of $f$ and its derivatives.)

(f) $\int_0^1 (t^2 \times (t^2 i + e^{-t^2} j + (\tan t) k)) \, dt$ (where $i$, $j$ and $k$ are the standard basis vectors in $\mathbb{R}^3$).

2. (34 points) (a) (20 points) Let $f$ be a positive continuous real-valued function on the interval $[-\pi/4, \pi/4]$. Let $A$ denote the area between the curve whose expression in polar coordinates is $r = f(\theta)$ ($-\pi/4 \leq \theta \leq \pi/4$) and the two lines $\theta = -\pi/4$ and $\theta = \pi/4$. Let $B$ denote the area between the curve whose expression in polar coordinates is $r = f(\theta/2)$ ($-\pi/2 \leq \theta \leq \pi/2$) and the vertical axis $\theta = \pm \pi/2$. Show that $B = 2A$. You may assume area formulas given in Stewart.

(b) (14 points) Find the area between the $y$-axis and the curve whose expression in polar coordinates is $r = \sec \theta/2$. You may use the result of part (a) whether or not you have proved it; or you may use any other method that gives the correct answer.

3. (12 points) Find equations in Cartesian (i.e., $(x, y, z)$) and spherical coordinates for the surface described in cylindrical coordinates by the equation $r^2 = z^2 + 1$. 