1. (20%) Let \( \alpha : [a, b] \to \mathbb{R}^3 \) be a regular curve. Show that there exists a function \( h : [c, d] \to [a, b] \) so that the curve \( \beta = \alpha \circ h : [c, d] \to \mathbb{R}^3 \) is a unit speed curve.

2. (15%) If \( \gamma \) is a unit speed curve in \( \mathbb{R}^3 \), compute \( \langle \gamma' \times \gamma'', \gamma''' \rangle \) in terms of curvature and torsion.

3. (15%) Prove that a unit speed curve with zero torsion is a plane curve.

4. (15%) Let \( \gamma \) be a unit speed curve on the sphere of radius \( r \) centered at a point \( \hat{m} \). Prove that the normal curvature of \( \gamma \) is constant. Explain clearly.

5. (15%) Let \( \tilde{z} : \mathbb{R}^2 \to \mathbb{R}^3 \) be the map \( \tilde{z}(u, v) = (u+v, u-v, uv) \). Show that \( \tilde{z} \) is a coordinate patch, and describe the surface \( \tilde{z}(\mathbb{R}^2) \).

6. (20%) Let \( \alpha : \mathbb{R} \to \mathbb{R}^3 \) be a simple, unit speed curve and let \( \alpha(s) = (\alpha_1(s), \alpha_2(s)) \). Let \( M \) be the surface \( \{ (\alpha_1(s), \alpha_2(s), t) : s, t \in \mathbb{R} \} \). Describe all the geodesics on \( M \), and supply detailed explanations.