Nonperturbative analysis of quasiperiodic operators

Svetlana Jitomirskaya, UC, Irvine

ABSTRACT

Study of fine spectral properties of quasiperiodic operators involves dealing with problems caused by small denominators, and until recently was only possible using perturbative methods, requiring certain small parameters and complicated KAM-type schemes. I will describe recently developed nonperturbative methods for such study which lead to stronger results and are significantly simpler.

A particularly important role is played here by the Lyapunov exponents that measure the exponential rate of growth of the transfer-matrices. Positivity of the Lyapunov exponents is known to imply a.e. absence of absolutely continuous spectrum, but does not lead in general to pure point spectrum, or localization. It turns out that it is possible to explore the structure of the transfer-matrices and analytic properties of the potential to extract more information out of positivity of the Lyapunov exponents.

The nonperturbative methods based on Lyapunov exponents and transfer-matrices have, however, certain limitations in that they, unlike the perturbative methods, do not extend well beyond one-dimensional, nearest-neighbor-interaction models. This was addressed recently through the development of non-transfer-matrix-based nonperturbative methods, which will also be discussed, as well as some applications.