Solitons, Particles and Noncommutative Geometry

ABSTRACT

One of the more puzzling discoveries in the theory of integrable systems is that the motion of the poles of meromorphic solutions to soliton equations (PDE such as the KP hierarchy) is often governed by integrable many-body systems (ODE such as the Calogero-Moser system). I will present an explanation of this phenomenon (joint work with T. Nevins) using (noncommutative) algebraic geometry. We study the space of "configurations of points on the quantum plane" and other spaces of noncommutative vector bundles as a natural bridge between solitons and particles. Namely, the soliton equations are realized as flows on these configurations, and a geometric Fourier transform converts the flows into the linear flows along tori (Jacobians of spectral curves) which give the "integration" of the many-body system.