Quantum and classical resonances for strongly trapping systems

Plamen Stefanov

Purdue University and MSRI

ABSTRACT

Resonances are one of the most studied objects in physics, and in quantum mechanics they describe states which in addition to a rest energy have a rate of decay. A mathematical example is given by the zeros of the Riemann zeta function which are the resonances of the quantum Hamiltonian given by the Laplacian on the modular surface. The term "resonance" comes from the analogy with "a bell sounding its dying notes" (a quote from Chandrasekhar describing resonances, or quasi-normal modes, of black holes). For a beautiful rendition of a recently constructed quantum resonator visit the Resonance Fine Art Gallery.

In this talk I will give a general introduction to the mathematical theory of resonances, following the approach introduced by Lax and Phillips. I will then present the recent progress in verifying the Lax-Phillips Conjecture in some cases and proving sharp lower and upper bounds on the density of resonances near the real axis. Those bounds are close in spirit to the classical Weyl asymptotics of the number of eigenvalues in a bounded domain. In the case of unbounded domains that we study, the leading term is connected to the measure of the trapped orbits only, instead of all rays as in the Weyl asymptotics.