January 27 William Stein, Harvard University

*Visibility of Shafarevich-Tate Groups of Modular Abelian Varieties at Higher Level*

I will begin by introducing the Birch and Swinnerton-Dyer conjecture in the context of abelian varieties attached to modular forms, and discuss some of the main results about it. I will then introduce Mazur’s notion of visibility of Shafarevich-Tate groups and explain some of the basic facts and theorems. Cremona, Mazur, Agashe, and myself carried out large computations about visibility for modular abelian varieties of level $N$ in $J_0(N)$. These computations addressed the following question: If $A$ is a modular abelian variety of level $N$, how much of the Shafarevich-Tate group of $A$ is modular of level $N$, i.e., visible in $J_0(N)$. The results of these computations suggest that often much of the Shafarevich-Tate group is not modular of level $N$. This suggests asking if every element is modular of level $N \ast m$, for some auxiliary integer $m$, and if so, what can one say about the set of such $m$? I will finish the talk with some new data and thoughts about this last question, which is still very much open.