On solvability and attractors of the Navier-Stokes equations

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Abstract

I deal with the systems

\[ u_t + u \cdot \nabla u - \text{div}\sigma(\varepsilon(v)) + \nabla \rho = f, \quad \text{div} v = 0, \tag{1} \]

where

\[ \sigma(\varepsilon) = \frac{\partial D(\varepsilon)}{\partial \varepsilon}, \tag{2} \]

and \( D \) is a given function characterizing the (incompressible) fluid.

For

\[ D(\varepsilon) = 2\nu\varepsilon \tag{3} \]

with \( \nu = \text{const} > 0 \), (1) is the Navier-Stokes system, describing the dynamics of Newtonian fluids when gradients of the velocity field \( u \) are “not large”. For the so-called generalized Newtonian fluids, \( \nu \) in (3) is a function of \( |\varepsilon| \).

I present the results about solvability of some initial-boundary value problems for (1), (2), the behavior of their solutions when \( t \to \infty \), and the estimation of the fractal dimension of the minimal global \( B \)-attractors for these problems.