

``Diffeomorphisms with robust statistical properties''

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Ergodicity is a basic property of conservative dynamical systems, first introduced by Boltzmann in his study of gases. A system is *ergodic* if it has no invariant sets, except those of measure 0 or of full measure.

Volume-preserving hyperbolic dynamical systems such as the toral map $(x,y) \mapsto (x + y, x + 2y)$ have robust statistical properties. In particular, such systems are *stably ergodic*: any sufficiently smooth, volume-preserving approximation of the original system is also ergodic. The stable ergodicity of hyperbolic systems was established by Anosov in the 1960's.

By contrast, non-hyperbolic systems can fail to be ergodic, and can even be stably non-ergodic. Until 5 years ago, there were no known examples of stably ergodic systems, other than the hyperbolic systems studied by Anosov. The theme of recent work of Grayson, Pugh, Shub, myself and others has been that *a little* hyperbolicity can imply stable ergodicity. I will discuss this phenomenon of stable ergodicity in non-hyperbolic systems.