In this lecture, I shall give an elementary and self-contained introduction to Dwork's conjecture (1973) on the p-adic meromorphic continuation of his unit root L-function $L(f,T)$ attached to a family $f: Y \to X$ of algebraic varieties over a finite field of characteristic $p$. This conjecture is a p-adic extension of the Weil conjecture (1949) from a single variety (or a family of zero-dimensional varieties) to an arbitrary family of varieties.

When $f$ is a family of zero-dimensional varieties (i.e., $f$ is a finite map), the unit root L-function $L(f, T)$ becomes the zeta function $Z(Y,T)$ of the variety $Y$. The zeta function $Z(Y,T)$ is a generating function which counts the number of rational points on $Y$. The zeta function $Z(Y,T)$ is a rational function as conjectured by Weil and first proved by Dwork (1960) using p-adic analytic method. The key step in Dwork's proof is to show that $Z(Y,T)$ is p-adic meromorphic. The zeros and poles of the zeta function satisfy a suitable Riemann hypothesis as also conjectured by Weil but proved by Deligne (1974-1980) using etale cohomology.

When $f$ is a family of positive-dimensional varieties, the unit root L-function $L(f, T)$ is no-longer a rational function and the situation is much more mysterious. But Dwork conjectured that the L-function is p-adic meromorphic. The simplest example is the universal family $f_E$ of elliptic curves. In this case, the L-function $L(f_E,T)$ is known to be p-adic meromorphic (Dwork, 1971). However, even in the elliptic family case, very little is known about the absolute values of the zeros of $L(f_E,T)$, namely, the p-adic Riemann hypothesis for $L(f_E,T)$, which contains important arithmetic information about modular forms such as the Gouvea-Mazur conjecture and the p-adic Ramanujan-Peterson conjecture.

![A note from the colloquium chair: I highly recommend the Dwork memorial article which was written for the March, 1999 Notices by Nick Katz and John Tate.](http://math.berkeley.edu/~ribet/Colloquium/dwan.html)